Sensor and Simulation Notes

Note 564

December 2014

A Power Wave Theory of Antennas
(Fourth Revision)

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Abstract

We introduce here a simple theory of antenna radiation and scattering that fully extends into the time domain a number of standard antenna terms, including gain, realized gain, effective length, antenna pattern, beamwidth, scattering cross section, and radar cross section. Power wave theory applies to linear reciprocal antennas of all feed impedances and feed types, including waveguide feeds. It also applies to antennas embedded in any lossless medium. The theory also leads to a natural definition of mutual coupling coefficient in antenna arrays. The approach is analogous to that used to describe circuits with generalized scattering parameters, with different reference impedances at each port. We identify receiving and transmitting impulse responses, and prove that they always have a simple relationship to each other, provided that the antenna has no nonlinear or nonreciprocal components. We also identify a scattering impulse response that can be applied to either an antenna or an arbitrary scatterer. From these functions, we build a Generalized Antenna Scattering Matrix (GASM), which provides a complete description of antenna response in the far field. This establishes a formalism that allows one to calculate antenna response under a variety of conditions, including, for example, a source or load of arbitrary impedance. The approach simplifies and clarifies terminology for characterizing antenna performance in both the time and frequency domains.

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I. Introduction

We address here the problem of characterizing antenna performance in the time domain. Currently, no standard terms have been defined in the antenna definitions standard [1], which becomes a challenge, for example, when buyers and sellers of wideband antennas need to discuss antenna specifications. In this work, we cast the antenna equations into a particularly simple form. This allows us to very naturally extend into the time domain a collection of commonly used antenna terms, including gain, realized gain, effective length, antenna pattern, beamwidth, radar cross section, and scattering cross section. Our formulation also clarifies antenna theory in the frequency domain, because it leads to a rigorous definition of mutual coupling coefficient in antenna arrays.

This work extends our earlier work [2, 3] and that of Baum [4]. We simplify Baum's antenna equations by normalizing the voltages and electric fields to the square root of a local reference impedance (typically 50 Ω or 120π Ω). This leads to obvious choices for antenna impulse response and scattering impulse response. We show how these functions are simply related to most of the commonly used expressions in antenna theory. Furthermore, we show how the resulting definitions can treat sources and loads with arbitrary impedances, and waveguide feeds. This covers nearly all the cases one can imagine for linear antennas without nonreciprocal components, such as ferrites. We limit our treatment here to antennas embedded in in lossless materials. While we do not treat lossy materials, neither do the standard definitions of antenna gain or radar cross section. (If they did, they would be dependent upon the distance of the observer from the antenna, which is not the case.) Lossy materials may be considered in a later paper.

The problem of antenna characterization in the time domain has been studied many times [4-15, 39-40]. This problem is related to that of optimizing antenna response in the time domain to a specified figure of merit [16-19]. Despite the large amount of previous work, there is no widely accepted method of characterizing antennas in the time domain. This is made clear by the fact that there are no definitions that describe time domain antenna performance in [1].

To illustrate the problem, we note how one might describe in the time domain the far-field performance of an Impulse Radiating Antenna (Figure 1.1) on boresight for dominant polarization. (The definition of the far field in the time domain is provided at the end of Section II.) One might show the voltage received into a 50-Ω load, when excited by an infinite plane-wave electric field with Gaussian time dependence (Figure 1.2, top). Alternatively, one might show the radiated electric field when driven by a Gaussian 50-Ω voltage source (bottom). One would normally show these for varying pulse widths (left and right). The bottom two waveforms are proportional to the derivatives of the top two waveforms. It is unnecessarily cumbersome to use four (or more) waveforms to describe the same phenomenon, when a single one will do. A primary goal of this paper is to reduce this description of antenna performance to a single waveform, the impulse response of an antenna.

We begin by deriving the antenna equations in transmission and reception, as described previously by Baum [4]. By using a different normalization, these expressions simplify considerably. This leads to obvious choices for impulse responses in transmission and reception, which are always related to each other by the antenna self-reciprocity equation. We then show how the impulse responses are related to common terms, such as gain, realized gain, and effective length. Next, we establish a scattering impulse response, which is closely related to radar cross section. The resulting parameters are then incorporated into a Generalized Antenna Scattering Matrix (GASM), which is reminiscent of scattering parameters in circuit theory. They are also somewhat reminiscent of Plane Wave Scattering Matrix Theory [36-38]. The GASM fully characterizes in the far field any linear reciprocal antenna in lossless media, in both the frequency and time domains. We show how the GASM is used to treat various problems associated with arbitrary impedances in the source or load. Finally, we suggest a number of standard terms that should be considered for inclusion in the antenna definitions standard [1].

We begin now by deriving the far-field antenna equations in transmission and reception.

Figure 1.1. The Farr Fields model IRA-3Q, with a diameter of 46 cm.

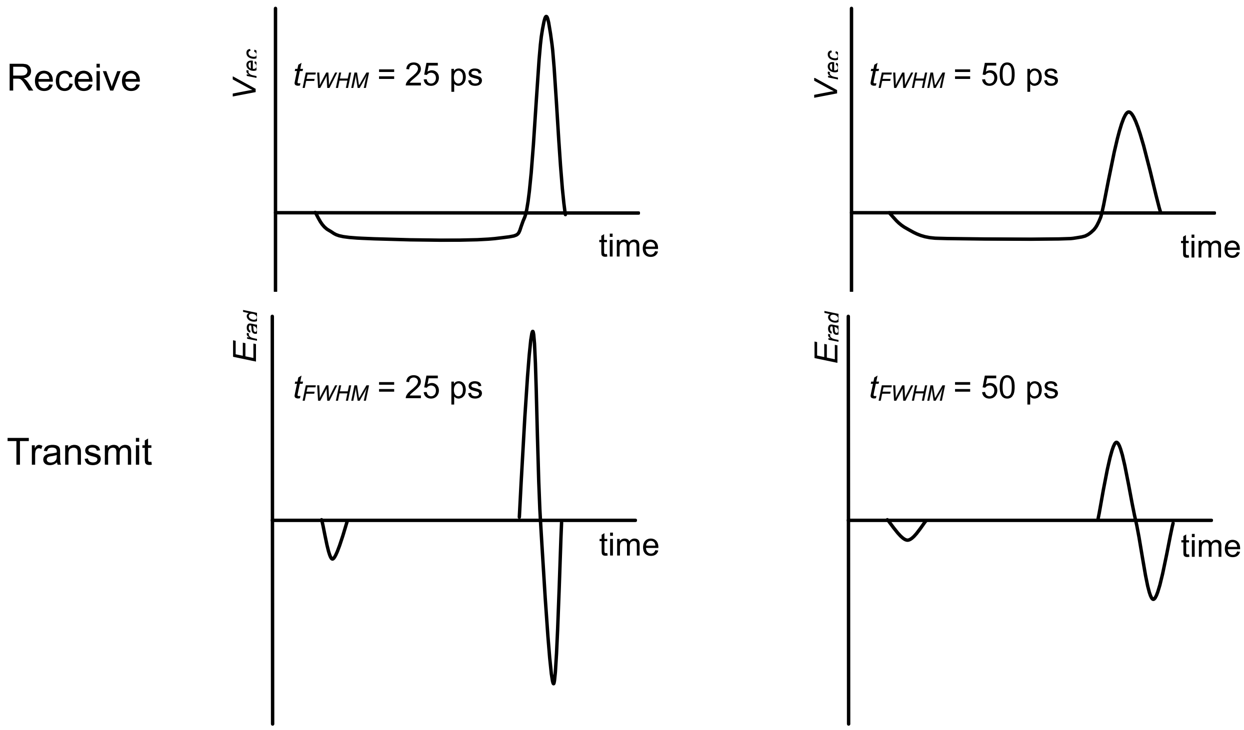

Figure 1.2. Characterizing an Impulse Radiating Antenna on boresight for dominant polarization by its response to an incident infinite plane-wave electric field with Gaussian time dependence (top), and a Gaussian 50-Ω source voltage (bottom), at two different Gaussian pulse widths (left and right). Note that t_{FWHM} is the Full-Width Half Max of the incident field or voltage. (Scale is approximate.)

II. Antenna Impulse Response in Transmission and Reception

We derive here the basic equations of transmission and reception in antennas in the far field, following the development of Baum [4]. We extend his work by normalizing voltages and fields to the square root of a local reference impedance. This simplifies the equations to the point where it becomes obvious how to define the impulse response of an antenna.

Before deriving the equations, we introduce the concept of reference impedance. Impulse response will be defined with respect to two reference impedances – one at the input port and one that is the characteristic impedance of the medium. These values are most commonly $Z_{o1} = 50\ \Omega$ and $Z_{o2} = 120\ \pi\ \Omega$, although they may both be any real value. In Section III, we show how such an impulse response can be extended to the case of waveguide feeds, where there is no obvious choice for the port reference impedance.

Real reference impedances impose only a small limitation on the types of problem that can be described. The S_{11} at the input port any circuit or antenna is always measured with respect to a real reference impedance, typically 50 Ω. Similarly, antenna gain, realized gain, and radar cross section are all defined only for lossless media. If lossy media were permitted, then these quantities would all vary with the distance of an observer from the antenna. There is considerable merit to extending this work to lossy media, but we leave that for a future paper.

In this section we treat only the dominant polarization of the antenna, looking only on boresight. Furthermore, the impedances of all sources and loads are equal to the port reference impedance. By studying just this extremely limited case, the definitions of most of the common antenna terms become apparent. At first, this might seem surprising. However, in an analogous way, 2-port circuits can also be fully characterized when both ports are terminated in 50 Ω sources or loads. In later sections we remove all restrictions and treat the most general case.

We derive all equations in the frequency domain, and only later convert to the time domain. We treat the frequency domain inversion as an inverse Laplace transform, so we use the frequency variable $s = j\omega$, where $\omega = 2\ \pi f$. Using the one-sided Laplace domain permits an added level of flexibility over the Fourier domain, since it can treat a number of additional cases. It can handle source signals that are not integrable, such as a step function. It can also treat initial conditions on the source or load, such as an initial voltage on a capacitor. Finally, it can treat complex frequencies. An excellent survey of the differences between the two domains is provided by Tesche and Bertholet in [41]. Note that a Fourier transform works if none of the above special cases is being analyzed.

We next introduce the concept of power waves. In the theory of generalized scattering parameters [22, p. 204], it is common to treat incident and scattered "waves," typically designated by $a_1 = \widetilde{V}^+ / \sqrt{Z_{o1}}$ and $b_1 = \widetilde{V}^- / \sqrt{Z_{o1}}$, where Z_{o1} is a real reference impedance associated with Port 1. Various authors use a variety of terminology to describe these waves. Pozar [22, p. 204] uses simply "wave amplitude," Gonzales uses "normalized voltage" [42, p. 29]. Vendelin [43, p. 7] uses "power waves." Collin [21], Kurokawa [22], and Gonzales [42, p. 45] use "power waves" with a somewhat different meaning than Vendelin, in which the reference impedance is the real part of a complex source or load impedance. For this paper, we find no alternative to "power wave," because we require three related varieties -- power waves, power flux density waves, and radiation intensity waves. We will see later that only by tracing power waves through an antenna can we realize the simplest form of the antenna equations, eqns. (3.3) and (3.4). We therefore use "power waves" in the sense used by Vendelin.

We consider first the equations of an antenna in transmission. We consider three cases for driving the antenna: open circuit voltage, $\tilde{V}$, short circuit current, $\tilde{I}$, and a source power wave, $\tilde{V}_{src}/\sqrt{Z_{o1}}$, as shown on the left in Figure 2.1. In this latter case, the source voltage, $\tilde{V}_{src}$, is a voltage wave incident upon the port from an infinitely long lossless feed transmission line of real characteristic impedance Z_{o1}. The antenna radiates into a medium of real characteristic impedance Z_{o2} and real propagation velocity v. The input impedance of the antenna, $\tilde{Z}_{in}$, may be complex.

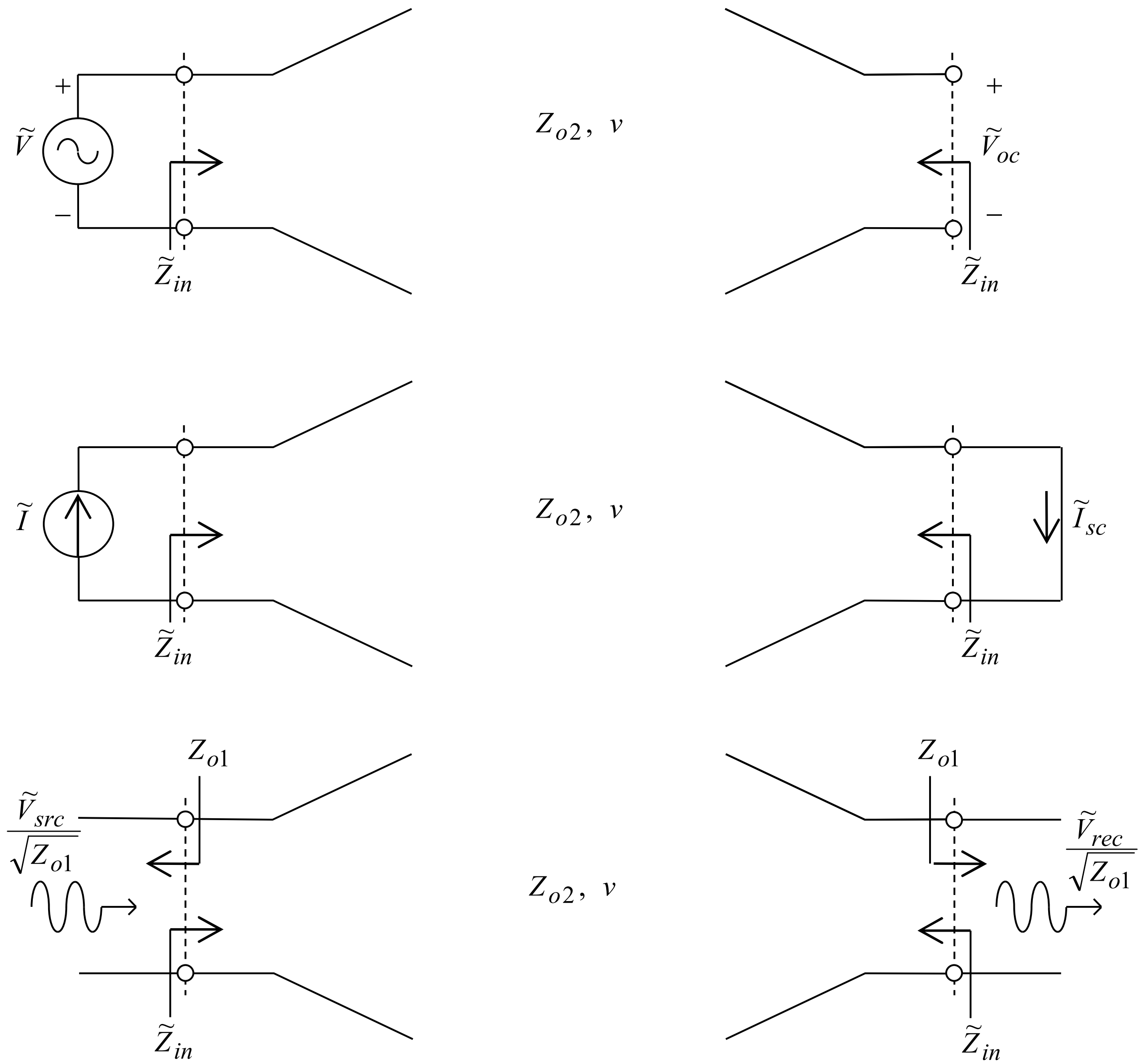

Figure 2.1. Three cases of antenna transmission (left) and reception (right) under conditions of open circuit (top), short circuit (middle), and power wave (bottom).

The equations in transmission may be expressed in one of three forms, depending on the source type: open circuit voltage, short circuit current, or power wave,

$$
\begin{aligned}
\tilde{E}_{rad} &= \frac{e^{-\gamma r}}{r} \tilde{F}_V \ \tilde{V} \\
&= \frac{e^{-\gamma r}}{r} \tilde{F}_I \ \tilde{I} \quad , \quad \tilde{F}_I = \tilde{Z}_{in} \ \tilde{F}_V \\
\frac{\tilde{E}_{rad}}{\sqrt{Z_{o2}}} &= \frac{e^{-\gamma r}}{r} \tilde{F} \ \frac{\tilde{V}_{src}}{\sqrt{Z_{o1}}}, \quad \tilde{F} = \sqrt{\frac{Z_{o1}}{Z_{o2}}} \frac{2\tilde{Z}_{in}}{\tilde{Z}_{in} + Z_{o1}} \tilde{F}_V = \sqrt{\frac{Z_{o1}}{Z_{o2}}} \frac{2}{\tilde{Z}_{in} + Z_{o1}} \tilde{F}_I
\end{aligned} \quad . \quad (2.1)
$$

Here, $\tilde{E}_{rad}$ is the radiated far field, $\gamma = s / v = jk$, $s = j\omega$, and $k = \omega / v = 2\pi f / v$ is the propagation constant in the surrounding medium. Note that these equations simply define the various forms of $\tilde{F}$, and show the relationship between them. We have intentionally normalized both sides of the third equation to the square root of the local reference impedance, for reasons that will become apparent shortly.

The power wave source may be envisioned as a Thévenin equivalent circuit with a voltage $2\tilde{V}_{src}$ and a impedance Z_{o1}, as shown in Figure 2.3. Sources with arbitrary impedances will be treated in Section VII.

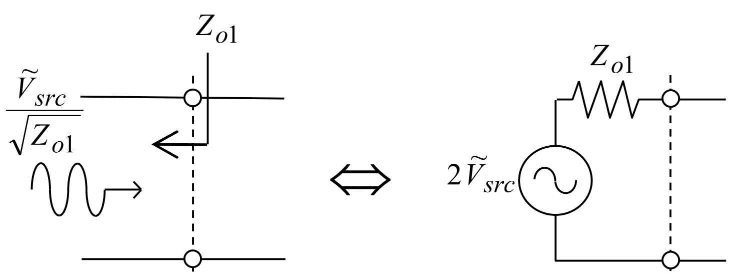

Figure 2.2. Thévenin equivalent circuit of the power wave source voltage.

Next, we consider the antenna equations in reception. As before, we consider three cases: open circuit voltage, short circuit current, and received power wave, as shown on the right in Figure 2.1. In the third case, the received voltage, $\tilde{V}_{rec}$, is the voltage wave launched onto an infinite lossless transmission line of real characteristic impedance Z_{o1}. Alternatively, $\tilde{V}_{rec}$ is just the received voltage across a load resistor of value Z_{o1}. In these cases we have

$$
\begin{aligned}
\tilde{V}_{oc} &= \tilde{h}_V \; \tilde{E}_{inc} \\
\tilde{I}_{sc} &= \tilde{h}_I \; \tilde{E}_{inc} \quad , \quad \tilde{h}_I = \frac{1}{\tilde{Z}_{in}} \tilde{h}_V \\
\frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} &= \tilde{h} \; \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} \quad , \quad \tilde{h} = \frac{Z_{o1}}{\tilde{Z}_{in} + Z_{o1}} \sqrt{\frac{Z_{o2}}{Z_{o1}}} \, \tilde{h}_V
\end{aligned}
\quad . \tag{2.2}
$$

Note that we use the convention here that positive current flows into the load. Note also that these equations simply define the various forms of $\tilde{h}$, and show the relationship between them.

We now seek a relationship between $\tilde{F}$ and $\tilde{h}$. To find this we use Baum's method [4], which applies the principle of reciprocity. Consider a two-port circuit consisting of two antennas positioned in each other's far field, as shown in Figure 2.3. If we consider this to be just two ports of a linear time-invariant circuit, then it can be described in terms of *Z*- or *Y*-parameters, which relate open circuit voltages on one port to short-circuit currents at the other. For a reciprocal system, $\tilde{Z}_{12} = \tilde{Z}_{21}$ and $\tilde{Y}_{12} = \tilde{Y}_{21}$, as shown, for example, in [22, pp. 193-194].

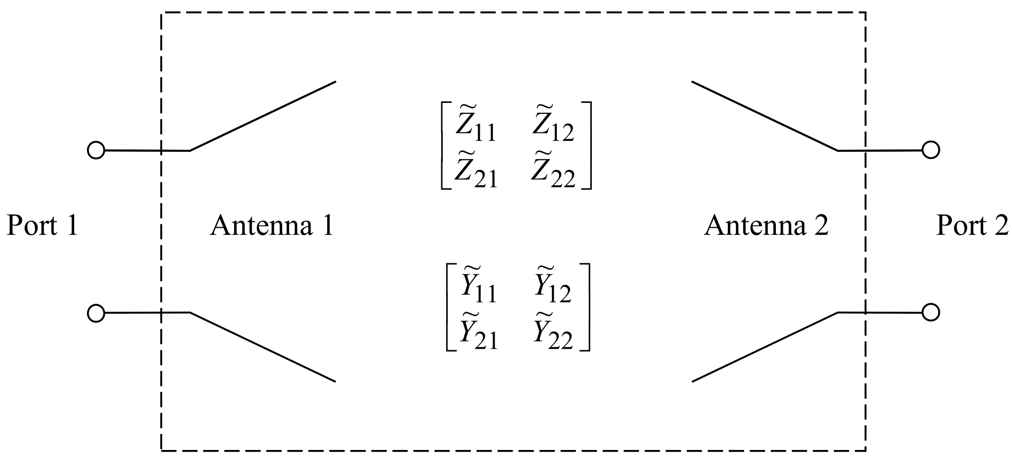

Figure 2.3. Two-port equivalent circuit of a two-antenna system.

Thus, if we drive Port 1 with an open circuit voltage and measure the short-circuit current at Port 2, we get the same result if we switch the two ports. The two short-circuit currents are expressed as

$$
\begin{aligned}
\tilde{I}_{sc}^{(2)} &= \frac{e^{-\gamma r}}{r} \tilde{h}_I^{(2)} \; \tilde{F}_V^{(1)} \; \tilde{V}^{(1)} \\
\tilde{I}_{sc}^{(1)} &= \frac{e^{-\gamma r}}{r} \tilde{h}_I^{(1)} \; \tilde{F}_V^{(2)} \; \tilde{V}^{(2)}
\end{aligned}
\quad , \tag{2.3}
$$

where the superscripts (1) and (2) specify the antenna. Since $\tilde{I}_{sc}^{(2)} = \tilde{I}_{sc}^{(1)}$ and $\tilde{V}^{(1)} = \tilde{V}^{(2)}$,

$$\frac{\widetilde{F}_V{}^{(1)}}{\widetilde{h}_I{}^{(1)}} = \frac{\widetilde{F}_V{}^{(2)}}{\widetilde{h}_I{}^{(2)}} \ , \tag{2.4}$$

and since $\widetilde{F}_V = \widetilde{F}_I \,/\, \widetilde{Z}_{in}$ and $\widetilde{h}_I = \widetilde{h}_V \,/\, \widetilde{Z}_{in}$, we have

$$\frac{\widetilde{F}_I{}^{(1)}}{\widetilde{h}_V{}^{(1)}} = \frac{\widetilde{F}_I{}^{(2)}}{\widetilde{h}_V{}^{(2)}} \, . \tag{2.5}$$

These are fundamental reciprocity relationships that must be satisfied by any linear reciprocal antenna.

The above equation demonstrates that the ratio $\widetilde{F}_I \,/\, \widetilde{h}_V$ is a universal constant associated with all linear reciprocal antennas. So if we know $\widetilde{F}_I \,/\, \widetilde{h}_V$ for a simple antenna, such as an electrically small electric or magnetic dipole (Antenna #2), we know the ratio for any arbitrary Antenna #1. Fortunately, both $\widetilde{F}_I$ and $\widetilde{h}_V$ have already been calculated for electrically small electric and magnetic dipoles, and we can take advantage of those results. For an electrically small electric dipole [4, 23]

$$\widetilde{F}_I = \frac{s\,\mu}{4\,\pi} h_e \ , \qquad \widetilde{h}_V = h_e \ , \tag{2.6}$$

where h_e is the effective height of the electrically small electric dipole and μ is the permeability of the surrounding medium, $\mu = Z_{o2} \,/\, v$. Note that [23] treats the case of arbitrary lossless media, while [4] is limited to dipoles in free space.

Similarly, for an electrically small magnetic dipole [4, 23]

$$\widetilde{F}_I = \frac{s^2 \mu}{4\,\pi\, v} A_h \ , \qquad \widetilde{h}_V = \frac{s\, A_h}{v} , \tag{2.7}$$

where A_h is the effective area of the magnetic dipole. Taking the ratios in the above two equations, we find for both electrically small electric and magnetic dipoles

$$\frac{\widetilde{F}_I}{\widetilde{h}_V} = \frac{s\,\mu}{4\,\pi} = \frac{\widetilde{F}_V}{\widetilde{h}_I} \, . \tag{2.8}$$

Because of the relationships in (2.4) and (2.5), these relationships must be true for all linear reciprocal antennas. We now calculate $\tilde{F}/\tilde{h}$ as

$$\frac{\tilde{F}}{\tilde{h}} = \frac{\tilde{F}}{\tilde{F}_I} \times \frac{\tilde{F}_I}{\tilde{h}_V} \times \frac{\tilde{h}_V}{\tilde{h}} . \tag{2.9}$$

By combining (2.9) with (2.1), (2.2), and (2.8), and using $v = Z_{o2} / \mu$, we obtain

$$\boxed{\tilde{F} = \frac{s\,\tilde{h}}{2\pi v} = \frac{j\,\tilde{h}}{\lambda}} . \tag{2.10}$$

This is the law of antenna self-reciprocity. For reasons that will become apparent later, we refer to $\tilde{F}$ as the transmitting transfer function, and $\tilde{h}$ as the receiving transfer function. The important result here is that once one knows an antenna's receiving transfer function, there is no need to further specify its transmitting transfer function. Of course, this expression only applies to those antennas operating in a linear fashion, without nonreciprocal components such as ferrites. This expression may also be found in [14, eqns. 4 & 5], where it is limited to antennas in free space.

In the time domain, the two transfer functions shown in eqn. (2.10), $\tilde{F}$ and $\tilde{h}$, become the transmitting and receiving impulse responses, respectively. Note that this terminology is a modification from earlier papers [2, 3, 7, 24], in which we used "impulse response" in both the time and frequency domains. Our new terminology is now consistent with common usage in system theory.

By combining (2.10) with (2.1) and (2.2), we find

$$\begin{aligned} \frac{\tilde{E}_{rad}}{\sqrt{Z_{o2}}} &= \frac{s}{2\pi v} \frac{e^{-\gamma r}}{r} \tilde{h} \frac{\tilde{V}_{src}}{\sqrt{Z_{o1}}} \\ \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} &= \tilde{h} \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} \end{aligned} . \tag{2.11}$$

In the most common case, $Z_{o1} = 50\Omega$, and $Z_{o2} = 120\pi\ \Omega$. This was the case that was treated in [2, 3], where we used the symbol $\tilde{h}_N$. Here, we use the symbol $\tilde{h}$ to represent a more general quantity.

If we take the inverse transform of the above equations, we obtain the results in the time domain,

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{Z_{o2}}} &= \frac{1}{2\pi v r}\, h(t) * \frac{d V_{src}(t')/dt}{\sqrt{Z_{o1}}} \qquad t' = t - r/v \\ \frac{V_{rec}(t)}{\sqrt{Z_{o1}}} &= h(t) * \frac{E_{inc}(t)}{\sqrt{Z_{o2}}} \end{aligned} \quad , \tag{2.12}$$

where "$*$" is the convolution operator.

Next, we derive an alternative form of the above results that will be more convenient in some cases. First, we note the general relationship,

$$f(t) * g'(t) = f'(t) * g(t), \tag{2.13}$$

where f and g are arbitrary functions of time, and the prime indicates a time derivative. Both $f(t)$ and $g(t)$ are assumed to be zero for $t < 0$. To prove this relationship, we express it in the frequency domain as

$$\tilde{f} \times s\,\tilde{g} = s\,\tilde{f} \times \tilde{g}, \tag{2.14}$$

where the relationship is now obvious. An alternative proof of 2.13 is found in [44], which states that $[f(t) * g(t)]' = f(t) * g'(t) = f'(t) * g(t)$. Note that some refinement may be required for non-zero initial conditions, which we leave for a later paper. Using this relationship, the transmitting equation becomes

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{Z_{o2}}} &= \frac{1}{2\pi v r}\, h'(t) * \frac{V_{src}(t')}{\sqrt{Z_{o1}}} \quad , \quad t' = t - r/v \\ \frac{V_{rec}(t)}{\sqrt{Z_{o1}}} &= h(t) * \frac{E_{inc}(t)}{\sqrt{Z_{o2}}} \end{aligned} \quad . \tag{2.15}$$

We will see that this form of the transmitting relationship can be more convenient in some cases.

Based on the above relationships, the law of antenna self-reciprocity, (2.10), can be represented in the time domain as

$$\boxed{F(t) = \frac{h'(t)}{2\pi v}} \,. \tag{2.16}$$

In other words, the transmitting impulse response of any linear antenna is simply the time derivative of the receiving impulse response divided by $2\pi v$. Recall that the physical meanings of

$F(t)$ and $h(t)$ are the inverse Laplace transforms of $\widetilde{F}$ and $\widetilde{h}$, which were defined earlier in eqns. (2.1) and (2.2), respectively.

Let us now consider how one could use the above equations on an antenna range, where there are two antennas. Combining the two equations for transmitting and receiving antennas in eqn. (2.15), we have

$$\frac{V_{rec}(t)}{\sqrt{Z_{o1}}} = \frac{1}{2\pi v r} \; h_{RX}(t) * h_{TX}(t) * \frac{d V_{src}(t') / dt}{\sqrt{Z_{o1}}} \qquad t' = t - r / v, \tag{2.17}$$

where $h_{TX}(t)$ and $h_{RX}(t)$ characterize the transmitting and receiving antennas, respectively.

The above equation suggests a simple method of characterizing an unknown antenna on an antenna range. One commonly calibrates the antenna range using two identical transmitting and receiving antennas. One then measures the source and receive voltages, and solves for the only remaining unknown, the $\widetilde{h}$ of the identical transmit/receive antennas, resulting in

$$\widetilde{h} = \sqrt{\frac{2\pi v r \; \widetilde{V}_{rec}}{s \, \widetilde{V}_{src}}} \, . \tag{2.18}$$

One then replaces one of the standard antennas with the antenna under test, and solves (2.17) for the unknown $\widetilde{h}_{AUT}$. Note that care must be taken to avoid dividing by a small number, and to choose the correct sign in the square root by unwrapping the phase. Additional detail is provided in [24, Appendix D].

The above equations are valid only for antennas that are in each other's far field. The far field of an antenna is that portion of the radiated field that is inversely proportional to the distance from the antenna, in either the frequency or time domain. The far field in the time domain is just the inverse transform of the far field in the frequency domain. When making measurements, one must set the antennas at a minimal distance from each other. In the frequency domain, this distance is described by [25, p. 30]

$$\begin{aligned} & r > 2D^2 / \lambda \\ & r >> \lambda \qquad , \\ & r >> D \end{aligned} \tag{2.19}$$

where D is the antenna diameter, and λ is the wavelength. Alternatively, Giri has formulated an analogous expression in the time domain [26]

$$r > D^2 / (2 c \, t_{mr}) , \tag{2.20}$$

where t_{mr} is the maximum source voltage divided by the maximum rate of rise. This is probably best understood as an estimate. In practice, one would normally verify that measured fields are indeed proportional to $1/r$ before accepting the data. This equation also appears in slightly different form in [27].

In the above equations we have assumed that both transmitting and receiving antennas are positioned so that reflections from the ground, chamber walls, and support structures are all excluded from $h(t)$ by using absorber material and/or with time gating.

Having expressed the field equations in the usual form, we now simplify those expressions further by using power waves, in the next section.

.

III. Antenna Equations Formulated With Power Waves

In [2, 3], we claimed that antenna equations similar to (2.11) and (2.12) were in the simplest possible form we could imagine. However, since writing those papers, we found two reasons why further simplification was necessary. First, it is challenging to express those equations in words, as is required by the antenna definitions standard [1]. Second, it is clumsy to use those expressions in signal flow graphs, which we use to simplify more complicated calculations, such as sources or loads with arbitrary impedances. We therefore found it necessary to simplify the antenna equations even further. As before, we limit our treatment to dominant polarization on boresight, with source and load impedances equal to the port reference impedance, Z_{o1}. A more general treatment is provided in Sections VI and VII.

To simplify eqns. (2.11) and (2.12), we introduce new terminology and symbols related to scattering parameters, which relate scattered power waves to incident power waves. To describe antennas, we require two additional types of power waves, a radiation intensity wave and a power flux density wave. The various power wave types are defined as follows:

$$
\begin{aligned}
\tilde{a}_1 &= \tilde{\Pi}_{src} = \frac{\tilde{V}_{src}}{\sqrt{Z_{o1}}} && = \text{source power wave} \\
\tilde{b}_1 &= \tilde{\Pi}_{rec} = \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} && = \text{received power wave} \\
\tilde{a}_2 &= \tilde{\Sigma}_{inc} = \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} && = \text{incident power flux density wave} \\
\tilde{b}_2 &= \tilde{Y}_{rad} = \frac{r\,\tilde{E}_{rad}}{\sqrt{Z_{o2}}} e^{\gamma r} && = \text{radiated radiation intensity wave}
\end{aligned}
\quad . \qquad (3.1)
$$

The symbols Π, Y, and Σ are Greek versions of P, U, and S, which are the commonly used symbols for power, radiation intensity, and power flux density, respectively [25, pp. 37-38]. Thus, to convert the symbol for a "power" quantity to that of a "power wave" quantity, we make the symbol Greek. To make the relationships clear, we note that these four power wave quantities are related to well-known quantities as

Power

$$\tilde{P}_{src} = \tfrac{1}{2}\,\mathrm{Re}\left(\tilde{V}_{src}\ \tilde{I}_{src}^{\ *}\right) = \left|\tilde{\Pi}_{src}\right|^2$$

$$\tilde{P}_{rec} = \tfrac{1}{2}\,\mathrm{Re}\left(\tilde{V}_{rec}\ \tilde{I}_{rec}^{\ *}\right) = \left|\tilde{\Pi}_{rec}\right|^2$$

Power Flux Density , (3.2)

$$\tilde{S}_{inc} = \tfrac{1}{2}\iint \mathrm{Re}\left(\tilde{\vec{E}}_{inc} \times \tilde{\vec{H}}_{inc}^{\ *}\right) \bullet d\vec{A} = \left|\tilde{\Sigma}_{inc}\right|^2$$

Radiation Intensity

$$\tilde{U}_{rad} = \tfrac{1}{2}\,\mathrm{Re}\left(\tilde{\vec{E}}_{rad} \times \tilde{\vec{H}}_{rad}^{\ *}\right) \bullet r^2 \hat{r} = \left|\tilde{Y}_{rad}\right|^2$$

where $\tilde{S}_{inc}$ is the incident power flux density on boresight and $\tilde{U}_{rad}$ is the radiated radiation intensity on boresight. Thus, we see that the various power wave quantities are simply the square roots of the corresponding power quantities, with a suitable phase.

Note that the various versions of power waves satisfy the laws of superposition, since they are just voltages or fields divided by the square root of a real impedance. If voltages or fields satisfy superposition, the same is true when then are multiplied by a scalar. Note also that the radiated radiation intensity wave, $\tilde{Y}_{rad}$, is independent of r as $r \to \infty$. Recall that $\tilde{E}_{rad} \propto 1/r$ in the far field, and the phase factor $e^{\gamma r}$ simply removes the time delay between the antenna and the observation point.

We now substitute the above expressions into eqn. (2.11) to find simpler versions of the transmit and receive equations in the frequency domain,

$$\begin{aligned} \tilde{Y}_{rad} &= \tilde{F}\ \tilde{\Pi}_{src} = \frac{s}{2\pi v}\tilde{h}\ \tilde{\Pi}_{src} = \frac{j}{\lambda}\tilde{h}\ \tilde{\Pi}_{src} \\ \tilde{\Pi}_{rec} &= \tilde{h}\ \tilde{\Sigma}_{\iota inc} \end{aligned} \quad . \qquad (3.3)$$

In the time domain, eqn. (2.12) becomes

$$\begin{aligned} Y_{rad}(t) &= F(t) * \Pi_{src}(t) = \frac{h'(t)}{2\pi v} * \Pi_{src}(t) = \frac{h(t)}{2\pi v} * \Pi'_{src}(t) \\ \Pi_{rec}(t) &= h(t) * \Sigma_{inc}(t) \end{aligned} \quad . \qquad (3.4)$$

These really do seem to be the simplest possible expressions for antenna radiation and reception. It will be straightforward to put these equations into words so they may be included in the antenna definitions standard [1]. Extensions to arbitrary angles and to two polarizations are handled in Section VI. Extensions to arbitrary source and load impedances are handled in Section VII.

Looking at the above equations, it now becomes apparent why $h(t)$ and $F(t)$ are referred to as the receiving and transmitting impulse responses, respectively. Similarly, it is also clear why $\tilde{h}$ and $\tilde{F}$ are referred to as the receiving and transmitting transfer functions. We note that the transmitting properties contain no new or unique information. To fully specify antenna performance, all that is necessary are the receiving properties, along with the port and medium reference impedances, and the velocity of propagation in the medium. For that reason, we recommend that $h(t)$ and $\tilde{h}$ may alternatively be called simply the "impulse response" and "transfer function" of the antenna, respectively. We explain the preference of $h(t)$ over $F(t)$ in Section VIII.

We can now appreciate the value of normalizing voltages and fields to the square root of the local reference impedances, Z_{o1} and Z_{o2}. This is the only way to force eqns. (3.3) and (3.4) to be independent of the reference impedances. This is also a well-known property of conventional scattering parameters [42, pp 35-36]. This property allows the equations to apply to waveguide feeds, for which there is no obvious choice of reference impedance. In this case, the reference impedance becomes a reference mode, for example, the TE_{10} mode of a WR-90 waveguide. While the impedance of a waveguide mode is ambiguous, the reflection coefficient at a port is well understood. These reflection coefficients will be used in Section VII to characterize antennas with arbitrary source and load impedances.

The new definitions of the various power wave quantities in eqn. (3.1) may at first glance look unfamiliar and unnecessary. However, we show in later sections how they are simply related to all of the familiar antenna terms in the frequency domain. Because they also form the basis of all the new time domain parameters, such as impulse response, the power waves defined in eqn. (3.1) begin to look like fundamental building blocks of antenna theory.

Next, we consider how to convert the transfer function of an antenna to commonly used terms in the frequency domain.

IV. Gain, Realized Gain, and Effective Length

We now derive the relationships of the antenna transfer function to realized gain, gain, and effective length. We derive realized gain from both the transmission and reception equations, and show that we obtain the same result. We follow the derivation in [2], but we have generalized it to cover more cases.

We continue to consider only dominant polarization on boresight, with source and load impedances equal to the port reference impedance, Z_{o1}. Even this limited case is sufficient for deriving the desired quantities. More general cases are treated in Sections VI and VII.

A. Realized Gain Derived from the Receive Equation

We begin by deriving realized gain from the receive equation. From (2.11) and (3.3), the received voltage into the reference impedance, Z_{o1}, is

$$\frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} = \tilde{h}\, \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} \quad , \qquad \tilde{\Pi}_{rec} = \tilde{h}\; \tilde{\Sigma}_{\iota inc}, \tag{4.1}$$

where we have used both sets of notation. If we multiply both sides of (4.1) by their complex conjugates, we get the received power as

$$\left| \tilde{P}_{rec} \right| = \left| \tilde{h} \right|^2 \left| \tilde{S}_{inc} \right|, \tag{4.2}$$

where $\tilde{S}_{inc}$ is the incident power flux density. Alternatively, the received power is

$$\left| \tilde{P}_{rec} \right| = \tilde{G}_r \frac{\lambda^2}{4\pi} \left| \tilde{S}_{inc} \right|, \tag{4.3}$$

where $\tilde{G}_r$ is the realized gain. If we now divide (4.3) by (4.2), we get

$$\tilde{G}_r = \frac{4\pi}{\lambda^2} |\, \tilde{h}\, |^2 = 4\pi \left| \tilde{F} \right|^2. \tag{4.4}$$

Note that $\tilde{h}$ and $\tilde{F}$ are dependent upon the reference impedances of both the port and the medium, typically 50 Ω and 120 π Ω, respectively. In addition, λ is the wavelength in the medium.

B. Realized Gain Derived from the Transmission Equation

Alternatively, we can derive realized gain from the transmitted field equation. In transmission, the radiated field is

$$\frac{\tilde{E}_{rad}}{\sqrt{Z_{o2}}} = \frac{s}{2\pi v}\frac{e^{-\gamma r}}{r}\tilde{h}\,\frac{\tilde{V}_{src}}{\sqrt{Z_{o1}}}\quad,\qquad \tilde{Y}_{rad} = \frac{j}{\lambda}\tilde{h}\,\tilde{\Pi}_{src}\ , \tag{4.5}$$

where $\gamma = jk$ and $s = j\,\omega$. If we multiply both sides by their complex conjugates, we get

$$\tilde{S}_{rad} = \frac{\left|\tilde{E}_{rad}\right|^2}{Z_{o2}} = \frac{1}{\lambda^2 r^2}\left|\tilde{h}\right|^2\frac{\left|\tilde{V}_{src}\right|^2}{Z_{o1}}\quad,\qquad \left|\tilde{Y}_{rad}\right|^2 = \frac{1}{\lambda^2}\left|\tilde{h}\right|^2\left|\tilde{\Pi}_{src}\right|^2\ . \tag{4.6}$$

where $\tilde{S}_{rad}$ is the radiated power density on boresight. Now realized gain is defined as

$$\tilde{G}_r = \frac{4\pi r^2\left|\tilde{S}_{rad}\right|}{\left|\tilde{P}_{src}\right|} = \frac{4\pi r^2\left|\tilde{E}_{rad}\right|^2/Z_{o2}}{\left|\tilde{V}_{src}\right|^2/Z_{o1}} = \frac{4\pi\left|\tilde{Y}_{rad}\right|^2}{\left|\tilde{\Pi}_{src}\right|^2} = \frac{4\pi\left|\tilde{h}\right|^2}{\lambda^2} = 4\pi\left|\tilde{F}\right|^2 . \tag{4.7}$$

This is the same result we found in (4.4). So the relationship between $\tilde{G}_r$ and $\tilde{h}$ is consistent when derived in two different ways.

C. Antenna Gain

Antenna gain may be found from realized gain using the relationship [1]

$$\tilde{G} = \frac{\tilde{G}_r}{1-\left|\tilde{\Gamma}\right|^2}\quad,\qquad \tilde{\Gamma} = \frac{\tilde{Z}_{in}-Z_{o1}}{\tilde{Z}_{in}+Z_{o1}}\ , \tag{4.8}$$

where $\tilde{\Gamma}$ is the reflection coefficient looking into the antenna port as measured relative to Z_{o1}. Note that the factor $1-|\tilde{\Gamma}|^2$ is the impedance mismatch factor, as defined in [1]. In the time domain, $\Gamma(t)$ is referred to as the reflection impulse response.

Realized gain is often a more useful measure of antenna performance than gain, because it includes the effect of impedance mismatch. It also simplifies the signal processing on a time domain antenna range, because no measurement of $\tilde{S}_{11}$ or $\tilde{\Gamma}$ is needed. For well-matched antennas, the two versions of gain are very close.

D. Effective Length

Effective length is the term in the antenna definitions standard [1] that is most closely related to the receiving transfer function, $\tilde{h}$. Effective length is the open circuit voltage in response to an incident plane wave. This is already defined in eqn. (2.2) as $\tilde{h}_V$, and is related to the transfer function by

$$\tilde{h}_V = \frac{\tilde{Z}_{in} + Z_{o1}}{Z_{o1}} \sqrt{\frac{Z_{o1}}{Z_{o2}}} \; \tilde{h} \; . \tag{4.11}$$

E. Effective Area

Let us consider now the effective area of an antenna. From [1, p. 13] and [25, p. 79], we have

$$A_e = \frac{\lambda^2}{4\pi} \tilde{G} \; . \tag{4.12}$$

Furthermore, from eqns. (4.8) and (4.4) we have

$$\tilde{G}_r = \tilde{G}\left(1 - \left|\tilde{\Gamma}\right|^2\right) = \frac{4\pi}{\lambda^2} \,|\, \tilde{h} \,|^2 \; . \tag{4.13}$$

Combining the above two equations, we have

$$A_e = \frac{\left|\tilde{h}\right|^2}{1 - \left|\tilde{\Gamma}\right|^2} \; . \tag{4.14}$$

So we see that the transfer function, $\tilde{h}$, is related to the square root of the effective area or aperture, except that it has a meaningful phase, and it is adjusted by the impedance mismatch factor.

V. Radar Cross Section, Scattering Cross Section, and the Scattering Impulse Response

The new expressions defined in Section III can be used to extend radar cross section (RCS) into the time domain. This idea can be applied either to an antenna, or just a simple scatterer. We treat here the case of dominant polarization on antenna boresight. The antenna port is assumed to be loaded with the port reference impedance, Z_{o1}. We treat more general cases later.

The incident power flux density wave is related to the radiation intensity wave as

$$\frac{\widetilde{E}_{rad}}{\sqrt{Z_{o2}}} = \widetilde{\ell}\ \frac{e^{-\gamma r}}{r} \frac{\widetilde{E}_{inc}}{\sqrt{Z_{o2}}} \quad , \qquad \widetilde{Y}_{rad} = \widetilde{\ell}\ \widetilde{\Sigma}_{inc} \ , \tag{5.1}$$

where $\widetilde{\ell}$ is the scattering coefficient and $\gamma = jk$. Here, we express the relationship using both sets of notation. In the time domain, $\ell(t)$ is the scattering impulse response. We can multiply (5.1) by its complex conjugate to get

$$r^2 \left| \widetilde{E}_{rad} \right|^2 = \left| \widetilde{\ell} \right|^2 \left| \widetilde{E}_{inc} \right|^2 \quad , \qquad \left| \widetilde{Y}_{rad} \right|^2 = \left| \widetilde{\ell} \right|^2 \left| \widetilde{\Sigma}_{inc} \right|^2 . \tag{5.2}$$

The usual expression for RCS is

$$\sigma = 4\pi\, r^2 \frac{\left| \widetilde{E}_{rad} \right|^2}{\left| \widetilde{E}_{inc} \right|^2} = 4\pi \frac{\left| \widetilde{Y}_{rad} \right|^2}{\left| \widetilde{\Sigma}_{inc} \right|^2} . \tag{5.3}$$

So we find the monostatic RCS on antenna boresight is just

$$\sigma = 4\pi \left| \widetilde{\ell} \right|^2 . \tag{5.4}$$

The scattering cross section generalizes the above to multiple polarizations and arbitrary angles of incidence and observation. We treat this in the next section. We treat the case of scattering when the antenna has an arbitrary load in Section VII.C.

VI. The Generalized Antenna Scattering Matrix and the Antenna Equation

We are now ready to define the Generalized Antenna Scattering Matrix (GASM), which is a complete far-field characterization of any linear reciprocal antenna embedded in a lossless medium. This aids our understanding of the equations, and it allows one to easily calculate antenna response when, for example, the source or load impedance is different than the port reference impedance, Z_{o1}. First, we treat the special case of dominant polarization on boresight. Then we extend it to two polarizations and arbitrary angles of incidence and observation.

To build the GASM, we combine the equations of Sections III and V into a set of matrix equations. We can express these two different ways, leading to

$$\begin{aligned} \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{bmatrix} &= \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{21} & \tilde{S}_{22} \end{bmatrix} \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{bmatrix} \\ \begin{bmatrix} \tilde{\Pi}_{rec} \\ \tilde{Y}_{rad} \end{bmatrix} &= \begin{bmatrix} \tilde{\Gamma} & \tilde{h} \\ s\,\tilde{h}/(2\pi v) & \tilde{\ell} \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\Sigma}_{inc} \end{bmatrix} \end{aligned} . \tag{6.1}$$

We sketch these in Figure 6.1, where the analogy to a 2-port network is clear. Note that Port 2 is a virtual port or radiation port. An analogous treatment of an antenna as a two-port device appears in [28].

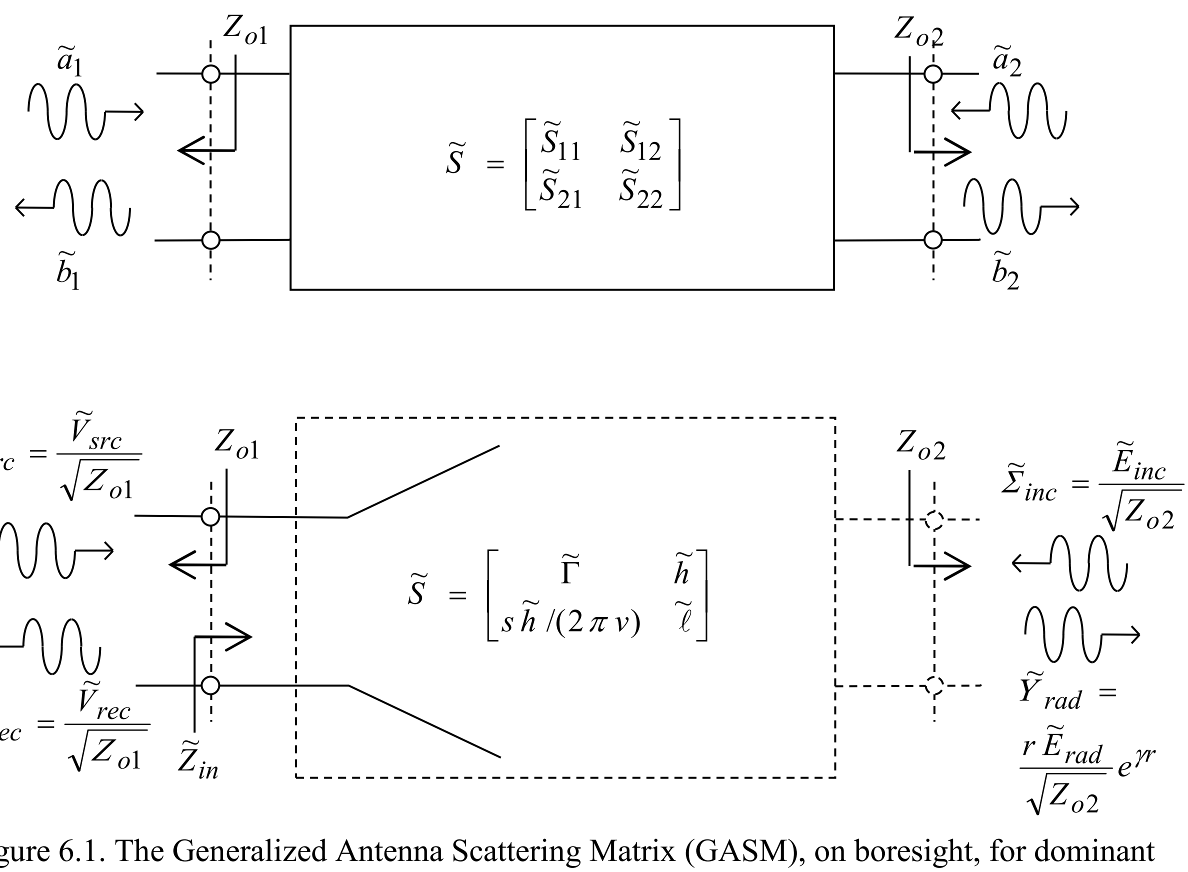

Figure 6.1. The Generalized Antenna Scattering Matrix (GASM), on boresight, for dominant polarization.

We find it useful to refer to eqn (6.1) as the antenna equation for dominant polarization on boresight.

The port voltage and current, $\widetilde{V}_p$ and $\widetilde{I}_p$, are proportional to the sum and difference of the two power waves at Port 1,

$$\begin{aligned} \widetilde{V}_p &= \left[\widetilde{\Pi}_{src} + \widetilde{\Pi}_{rec} \right] \sqrt{Z_{o1}} \\ \widetilde{I}_p &= \left[-\widetilde{\Pi}_{src} + \widetilde{\Pi}_{rec} \right] \Big/ \sqrt{Z_{o1}} \end{aligned} , \tag{6.2}$$

where $\widetilde{I}_p$ is the current flowing into the load.

It may be useful to compare eqn. (6.1) above to eqn. (1.3-11) in Kerns [36], since they look somewhat similar. Kerns's Plane Wave Scattering Matrix shows how waveguide modes at the antenna input port are related to a spatial spectrum of plane waves in the near field. Our GASM shows how power waves at the input port are related to the far fields for dominant polarization on boresight. Later in this section we generalize to both polarizations and arbitrary angles.

In the time domain the antenna equation, eqn. (6.1), takes the form

$$\begin{bmatrix} \Pi_{rec}(t) \\ Y_{rad}(t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h(t) \\ h'(t)/(2\pi v) & \ell(t) \end{bmatrix} \overset{*}{\bullet} \begin{bmatrix} \Pi_{src}(t) \\ \Sigma_{inc}(t) \end{bmatrix}, \tag{6.3}$$

where the " $\overset{*}{\bullet}$ " operator is a matrix-product convolution operator, defined as

$$\begin{bmatrix} s_{11}(t) & s_{12}(t) \\ s_{21}(t) & s_{22}(t) \end{bmatrix} \overset{*}{\bullet} \begin{bmatrix} a_1(t) \\ a_2(t) \end{bmatrix} = \begin{bmatrix} s_{11}(t) * a_1(t) + s_{12}(t) * a_2(t) \\ s_{21}(t) * a_1(t) + s_{22}(t) * a_2(t) \end{bmatrix}. \tag{6.4}$$

We were unable to find a standard notation for the above operation, so we defined our own.

The above representation is reminiscent of generalized scattering parameters [22, p. 204], which permit different reference impedances at each port. However, there are at least three major differences. First, the elements of the GASM are not unitless. Indeed, both $\widetilde{h}$ and $\widetilde{\ell}$ have units of meters. Second, it is never the case that $S_{12} = S_{21}$, which is normally a property of reciprocal devices operating in a linear fashion. In the GASM this can never be true because the units are mismatched. Finally, the GASM can never be unitary (determinant equal to unity), while this property is always true of the scattering matrices of lossless linear two-port devices. These facts will no doubt be disconcerting to some, but the value of the GASM lies in its ability to simplify and clarify the antenna equations.

It is useful now to represent the above equation as a signal flow graph, as shown in Figure 6.2. The theory of signal flow graphs is discussed in several texts [22, 29, 30, 42]. This formalism simplifies calculations involving sources or loads with arbitrary impedance, as described in the next section.

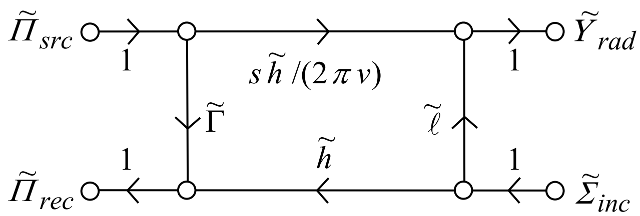

Figure 6.2 Signal flow graph representation of the GASM.

Next we extend the antenna equation to the more general case of two polarizations, with arbitrary angles of incidence and observation. We use a spherical coordinate system of (r, θ, ϕ) with $\theta = 0$ on antenna boresight. For now, we consider only linear polarization – we discuss circular polarization later. Thus, we have

$$\begin{bmatrix} \tilde{\Pi}_{rec}(\theta',\phi') \\ \tilde{Y}_{\theta,rad}(\theta,\phi,\theta',\phi') \\ \tilde{Y}_{\phi,rad}(\theta,\phi,\theta',\phi') \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h}_\theta(\theta',\phi') & \tilde{h}_\phi(\theta',\phi') \\ s\tilde{h}_\theta(\theta,\phi)/(2\pi v) & \tilde{\ell}_{\theta\theta}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{\theta\phi}(\theta,\phi,\theta',\phi') \\ s\tilde{h}_\phi(\theta,\phi)/(2\pi v) & \tilde{\ell}_{\phi\theta}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{\phi\phi}(\theta,\phi,\theta',\phi') \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\Sigma}_{\theta,inc}(\theta',\phi') \\ \tilde{\Sigma}_{\theta,inc}(\theta',\phi') \end{bmatrix}. \tag{6.5}$$

In this expression, the unprimed angular coordinates are the angles of observation, and the primed coordinates are the angles of incidence. In the time domain, the antenna equation becomes

$$\begin{bmatrix} \Pi_{rec}(\theta',\phi',t) \\ Y_{\theta,rad}(\theta,\phi,\theta',\phi',t) \\ Y_{\phi,rad}(\theta,\phi,\theta',\phi',t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h_\theta(\theta',\phi',t) & h_\phi(\theta',\phi',t) \\ h'_\theta(\theta,\phi,t)/(2\pi v) & \ell_{\theta\theta}(\theta,\phi,\theta',\phi',t) & \ell_{\theta\phi}(\theta,\phi,\theta',\phi',t) \\ h'_\phi(\theta,\phi,t)/(2\pi v) & \ell_{\phi\theta}(\theta,\phi,\theta',\phi',t) & \ell_{\phi\phi}(\theta,\phi,\theta',\phi',t) \end{bmatrix} \ast \begin{bmatrix} \Pi_{src}(t) \\ \Sigma_{\theta,inc}(\theta',\phi',t) \\ \Sigma_{\phi,inc}(\theta',\phi',t) \end{bmatrix} \tag{6.6}$$

So an antenna is completely specified by $\Gamma(t)$, two components of $\vec{h}(\theta,\phi,t)$, and. three components of the dyadic $\overleftrightarrow{\ell}(\theta,\phi,\theta',\phi't)$. (Note that $\ell_{\phi\theta}(\theta,\phi,\theta',\phi',t) = \ell_{\theta\phi}(\theta,\phi,\theta',\phi',t)$ in linear antennas.) Alternatively, an antenna could be specified in terms of the frequency domain versions of these quantities. All quantities assume a specified port reference impedance, Z_{o1}, a specified medium impedance, Z_{o2}, and a velocity of propagation through the medium, v.

A compact way of expressing (6.5) is to introduce vector and dyadic notation. Thus, we have

$$\begin{bmatrix} \tilde{\Pi}_{rec} \\ \tilde{\vec{Y}}_{rad} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{\vec{h}}^{\mathrm{T}} \\ s\,\tilde{\vec{h}}/(2\pi v) & \tilde{\overleftrightarrow{\ell}} \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\vec{\Sigma}}_{inc} \end{bmatrix}, \tag{6.7}$$

where the superscript "T" indicates a transposed vector. This more compact form may be represented in a signal flow graph as shown in Figure 6.3, where multiplications are now interpreted as matrix products. An alternative representation that avoids a vectorized signal flow graph is shown in Figure 6.4. This is a direct implementation of eqn. (6.5).

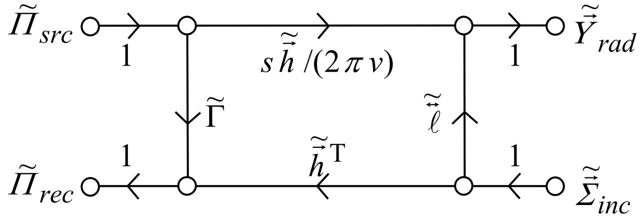

Figure 6.3 Signal flow graph representation of the GASM including both polarizations.

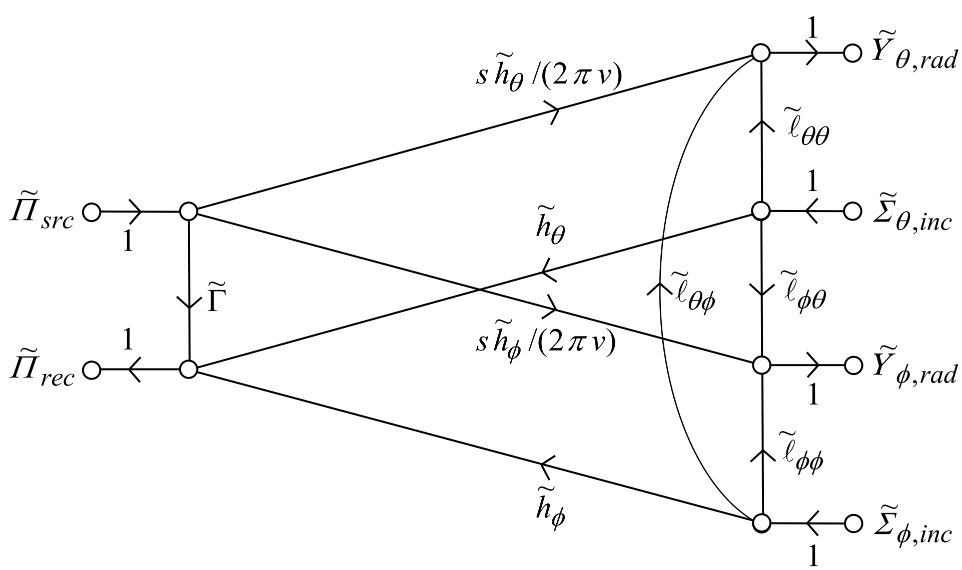

Figure 6.4 Scalar signal flow graph representation of the GASM including both polarizations.

In the next section we use signal flow graphs to calculate antenna responses with arbitrary source and load impedances.

VII. Arbitrary Source and Load Impedances

Until now, we have assumed that the source and load impedances were always equal to a real reference impedance, Z_{o1}. We treat here arbitrary source and load impedances using signal flow graphs, which are also commonly used with standard scattering parameters.

A. Transmitting with a Source of Arbitrary Impedance

We begin by calculating the radiated field when an antenna is driven by a source with arbitrary complex source impedance. At first, we look only on boresight, for dominant polarization. The goal is to find the ratio of $\tilde{Y}_{rad}$ to $\tilde{\Pi}_{src}$. The antenna is embedded in a medium of characteristic impedance Z_{o2}. A sketch of the relevant portion of the signal flow graph is shown in Figure 7.1. We are able to delete parts of it because there is no incident field (or incident power flux density wave).

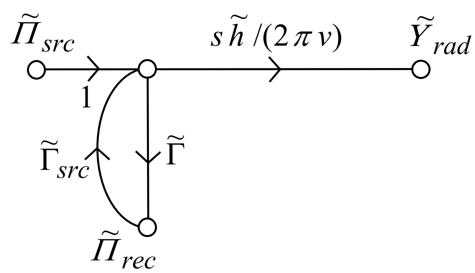

Figure 7.1 Signal flow graph for a source with arbitrary impedance.

The only new term is $\tilde{\Gamma}_{src}$, the reflection from the source impedance, given by

$$\tilde{\Gamma}_{src} = \frac{\tilde{Z}_{src} - Z_{o1}}{\tilde{Z}_{src} + Z_{o1}}. \tag{7.1}$$

The graph is now resolved as

$$\tilde{Y}_{rad} = \frac{1}{1 - \tilde{\Gamma}\,\tilde{\Gamma}_{src}} \frac{s\,\tilde{h}}{2\pi\nu}\,\tilde{\Pi}_{src}. \tag{7.2}$$

Signal flow graphs can be resolved using a standard set of four rules [30, 22, p. 214], or with Mason's rule [30, 42].

To find the required input parameters, $\tilde{\Gamma}_{src}$ and $\tilde{\Pi}_{src}$, we need to establish a relationship between a source power wave and a Thévenin equivalent source, which is probably more familiar to most readers. The result is shown in Figure 7.2, which assumes the reference impedance of the port is Z_{o1}.

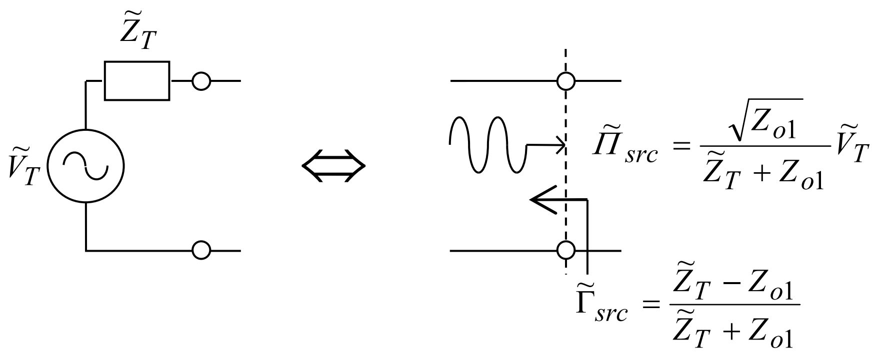

Figure 7.2. The relationship between a Thévenin equivalent source (left) and a power wave source (right) at a port with reference impedance Z_{o1}.

Note that the reference impedance of the port, Z_{o1}, does not appear explicitly in eqn. (7.2), except in the definitions of two reflection coefficients and the source power wave. So eqn. (7.2) can be used for waveguide feeds, for which reference impedances are ill-defined, but reflection coefficients are available.

To extend this result to both polarizations and arbitrary angles, one could build a diagram for each polarization. The result would be

$$\tilde{\vec{Y}}_{rad}(\theta,\phi) = \frac{1}{1-\tilde{\Gamma}\tilde{\Gamma}_{src}} \frac{s\,\tilde{\vec{h}}(\theta,\phi)}{2\pi v} \tilde{\Pi}_{src}. \tag{7.3}$$

This provides the complete solution for radiation with a source with arbitrary impedance. Conjugate match occurs when $\tilde{Z}_{src} = \tilde{Z}_{in}^{*}$, and hence $\tilde{\Gamma}_{src} = \tilde{\Gamma}^{*}$. This optimizes power transfer for an antenna with a given input reflection coefficient, as shown by Gonzalez [42, p. 240].

B. Receiving into an Arbitrary Load

Next, we consider antenna reception into an arbitrary complex load. As before, we begin by treating incidence on boresight, for dominant polarization. The goal is to find the ratio of $\tilde{\Pi}_{rec}$ to $\tilde{\Sigma}_{inc}$, and the port voltage. The antenna is embedded in a medium of impedance Z_{o2}. A sketch of the relevant portion of the signal flow graph is shown in Figure 7.3. We are able to delete parts of it because there is no source power wave driving the port.

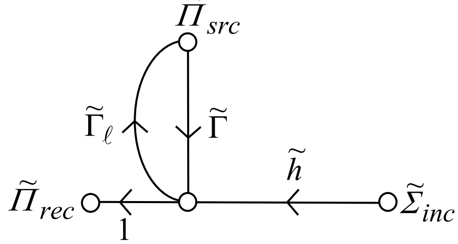

Figure 7.3 Signal flow graph for reception into an arbitrary load.

The new term is $\tilde{\Gamma}_{\ell}$, the reflection from the load impedance, given by

$$\tilde{\Gamma}_{\ell} = \frac{\tilde{Z}_{\ell} - Z_{o1}}{\tilde{Z}_{\ell} + Z_{o1}} . \tag{7.4}$$

The graph resolves as

$$\tilde{\Pi}_{rec} = \frac{1}{1 - \tilde{\Gamma}\,\tilde{\Gamma}_{\ell}} \, \tilde{h} \, \tilde{\Sigma}_{inc} . \tag{7.5}$$

This equation now takes into account arbitrary complex load impedance. The port voltage is found from the sum of two power waves, the received power wave, $\tilde{\Pi}_{rec}$, and the reflection from the load, $\tilde{\Pi}_{src} = \tilde{\Gamma}_{\ell}\,\tilde{\Pi}_{rec}$. Thus, from eqn. (6.2), the port voltage is

$$\tilde{V}_p = \sqrt{Z_{o1}} \, (1 + \tilde{\Gamma}_{\ell}) \, \tilde{\Pi}_{rec} . \tag{7.6}$$

To extend this result to both polarizations and arbitrary angles, one could build a diagram for each polarization and add the results to get

$$\tilde{\Pi}_{rec} = \frac{1}{1 - \tilde{\Gamma}\,\tilde{\Gamma}_{\ell}} \, \tilde{\tilde{h}}^T (\theta', \phi') \cdot \tilde{\tilde{\Sigma}}_{inc}(\theta', \phi') . \tag{7.7}$$

This provides the complete solution for reception into an arbitrary load. Conjugate match occurs when $\tilde{Z}_\ell = \tilde{Z}_{in}^{\;*}$, and hence $\tilde{\Gamma}_\ell = \tilde{\Gamma}^*$. This optimizes power transfer to the load for an antenna with a given input reflection coefficient, as shown by Gonzalez [42, p. 240].

C. Scattering from an Antenna with Arbitrary Load

Next, we consider the case of scattering from an antenna with an arbitrary complex load. The signal flow graph of the configuration is shown in Figure 7.4. We begin by treating incidence on boresight, for dominant polarization. The goal is to find the ratio of $\tilde{Y}_{rad}$ to $\tilde{\Sigma}_{inc}$. The antenna is embedded in a medium of impedance Z_{o2}.

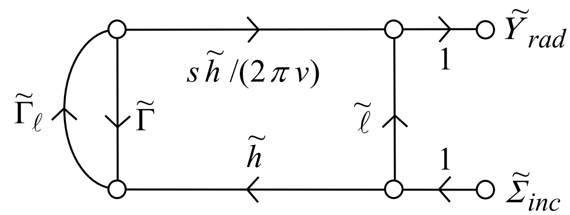

Figure 7.4 Scattering from an arbitrary load.

The signal flow graph resolves as

$$\tilde{Y}_{rad} = \left[\frac{\tilde{\Gamma}_\ell}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell} \frac{s\tilde{h}^2}{2\pi v} + \tilde{\ell} \right] \tilde{\Sigma}_{inc} . \tag{7.8}$$

One can find an example of how a similar graph is resolved in [22, p. 216]. When we generalize to two polarizations and arbitrary angles of incidence and observation we obtain.

$$\vec{\tilde{Y}}_{rad} = \left[\frac{\tilde{\Gamma}_\ell}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell} \frac{s}{2\pi v} \vec{\tilde{h}}(\theta,\phi)\, \vec{\tilde{h}}^T(\theta',\phi') + \overleftrightarrow{\tilde{\ell}}(\theta',\phi',\theta,\phi) \right] \cdot \vec{\tilde{\Sigma}}_{inc}(\theta',\phi') . \tag{7.9}$$

where the dyadic is calculated as

$$\begin{aligned} \vec{\tilde{h}}(\theta,\phi)\, \vec{\tilde{h}}^T(\theta',\phi') \cdot \vec{\tilde{\Sigma}}_{inc}(\theta',\phi') &= \vec{\tilde{h}}(\theta,\phi) \left\{ \vec{\tilde{h}}^T(\theta',\phi') \cdot \vec{\tilde{\Sigma}}_{inc}(\theta',\phi') \right\} \\ &= \vec{\tilde{h}}(\theta,\phi) \left\{ \tilde{h}_\theta(\theta',\phi')\, \tilde{\Sigma}_{\theta,inc}(\theta',\phi') + \tilde{h}_\phi(\theta',\phi')\, \tilde{\Sigma}_{\phi,inc}(\theta',\phi') \right\} . \end{aligned} \tag{7.10}$$

This provides the complete solution for scattering from an antenna with an arbitrary load.

VIII. Transmitting vs. Receiving Antenna Characteristics

At several points in this paper, we express a preference for characterizing an antenna by its receiving transfer function and impulse response ($\tilde{h}$ and $h(t)$) instead of its transmitting transfer function and impulse response ($\tilde{F}$ and $F(t)$). The transmitting characteristics are always simply related to the receiving characteristics, and, as a rule, one should not use two functions when a single one will do. For that reason, we recommend that "impulse response" and "transfer function," without qualifiers, refer specifically to the receiving characteristics. We explain here our preference of one over the other.

First, let us concede that the units of the two impulse responses favor designating the transmitting impulse response, $F(t)$, as the special one. The units of $F(t)$ and $\tilde{F}$ are 1/seconds and unitless, respectively. The units of $h(t)$ and $\tilde{h}$ are meters/second and meters, respectively. Other things being equal, we prefer a transfer function that is unitless in the frequency domain.

However, other things are not equal, and we prefer $h(t)$ over $F(t)$ for three specific reasons. First, the shape of $h(t)$ is simpler to interpret than that of $F(t)$. In certain types of wideband antennas, typically with focused apertures, $h(t)$ is an impulse-like function, which is relatively easy to look at and interpret. On the other hand, $F(t)$ is proportional to the derivative of $h(t)$, so it is like a doublet, which is more challenging to analyze and interpret.

The second reason for preferring $h(t)$ over $F(t)$ is related to the first reason, but in the frequency domain. If $h(t)$ is an impulse-like function, then its Fourier or Laplace transform is approximately flat over the frequency band. This is a convenient property for examining broadband data. On the other hand, the Fourier or Laplace transform of $F(t)$ increases with frequency at an approximate rate of 20 dB/decade. Such a waveform is harder to interpret, because it emphasizes the high-frequency part of the spectrum, and de-emphasizes the low-frequency part.

The third reason for preferring $h(t)$ over $F(t)$ concerns practical numerical considerations. Consider once again eqn. (2.17), which is the two-antenna range equation on boresight for dominant polarization. The first step in the analysis is to take the derivative of the source function, $V_{src}(t)$ and convert it to the frequency domain, possibly with a discrete Fourier transform. Typically, the source voltage is an approximate step- or impulse-function. The derivatives of these two functions are approximate impulse- or doublet-functions, both of which begin and end near zero, so the Fourier transform operation is well behaved.

On, the other hand, we can see a practical difficultly associated with using $F(t)$ by recasting eqn. (2.17) into an expression using $F(t)$,

$$V_{rec}(t) \;=\; \frac{2\pi v}{r}\; F_{RX}(t) * F_{TX}(t) * \int V_{src}(t')\, dt \qquad t' \;=\; t - r/v\,. \tag{8.1}$$

In this case, the first step in the analysis is to take an integral of the source voltage and transform it into the frequency domain. But the integrals of approximate step- or impulse-functions are approximate ramp- or step-functions. These functions are not easily transformed to the frequency domain, because they do not begin and end near zero. This is a problem because common signal analysis with Fourier transforms assumes that signals are periodic, which implies an abrupt discontinuity where the signal repeats. Discontinuous functions are difficult to model as a sum of sinusoids, so the use of $F(t)$ is numerically less stable. It may be possible to handle the problem using the Fast Laplace Transform [41], but this is still a harder problem to solve, and the added complexity offers no offsetting benefit.

Thus, for the three reasons stated above, we prefer to establish the receiving characteristics as the default for characterizing antennas.

IX. Related Issues

We consider here a number of additional issues related to antenna impulse response and power wave theory.

A. Antenna Arrays and Multimode Waveguide Feeds

If there is an array of N antennas, then eqn. (6.7) can be modified to accommodate this. The transfer function becomes a $2 \times N$ matrix (2 rows, N columns), $\tilde{h}_{i\,j}$, $i = 1,2;$ $j = 1,\ldots,N$; where i indicates one of two polarizations, and j indicates the port number. Furthermore, the source and received power waves become N-element vectors, $\tilde{\vec{\Pi}}_{src}$ and $\tilde{\vec{\Pi}}_{rec}$, and the input reflection coefficient becomes an $N \times N$ matrix, $\tilde{\tilde{\Gamma}}$. In this formulation, $\tilde{\Gamma}_{i\,j}$ represents the mutual coupling coefficient seen at port i from a source at port j. Each port is terminated in its own reference impedance, Z_{oj} , j=1,...,N; and the medium impedance is Z_{oN+1}. Under these conditions, the antenna equations become

$$\begin{bmatrix} \tilde{\vec{\Pi}}_{rec} \\ \tilde{\vec{Y}}_{rad} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{\Gamma}} & \tilde{\tilde{h}}^{\mathrm{T}} \\ s\,\tilde{\tilde{h}}\,/(2\pi\, v) & \tilde{\tilde{\ell}} \end{bmatrix} \begin{bmatrix} \tilde{\vec{\Pi}}_{src} \\ \tilde{\vec{\Sigma}}_{inc} \end{bmatrix}, \qquad (9.1)$$

where the matrix dimensions can be visualized as

$$\begin{bmatrix} [\quad]_{N\times 1} \\ [\quad]_{2\times 1} \end{bmatrix} = \begin{bmatrix} [\qquad]_{N\times N} & [\quad]_{N\times 2} \\ [\qquad]_{2\times N} & [\quad]_{2\times 2} \end{bmatrix} \begin{bmatrix} [\quad]_{N\times 1} \\ [\quad]_{2\times 1} \end{bmatrix}. \qquad (9.2)$$

One can measure $\tilde{\Gamma}_{i\,j}$, the mutual coupling coefficient seen at port i from a source at port j, by driving port j with a known source and measuring the response at port i, while terminating all other ports with their respective reference impedances or, in the case of waveguides, reference modes with no reflections. The antenna is assumed to be radiating into an infinite medium of impedance Z_{oN+1}. This is a clarification of frequency domain terminology, since mutual coupling is currently defined only in very general terms [1]. In the time domain, $\Gamma_{i\,j}(t)$ is referred to as the mutual coupling impulse response. When $i = j$, this is the reflection impulse response at the i^{th} port.

Multimode waveguide feeds are treated similarly. In this case, each mode is treated the same as an element in an antenna array. Mode magnitudes are normalized so they represent the square root of power in the mode.

B. TDR Response

The TDR (Time Domain Reflectometer) response of an antenna is a useful way of characterizing reflections from an antenna port. There are two ways of thinking about the TDR response. The first approach is to consider this as the raw response of a special oscilloscope with a TDR source, which sends an approximate step-function voltage out to the port. The risetime on the approximate step function must be fast enough to reveal interesting details in the antenna, and to cover the intended bandwidth. The return signal from the antenna is what one normally thinks of when referring to a TDR response. The challenge here is that different instruments provide different risetimes in their incident pulse, so the received waveform varies with the specific instrument used. We refer to this as the raw TDR response as measured on a specific instrument, $TDR_r(t)$.

A second approach to TDR response is simply to integrate the reflection impulse response with respect to time,

$$TDR_c(t) \;=\; \int_0^t \Gamma(t')\, dt' \,. \tag{9.3}$$

This expression has the advantage that it is independent of the specific instrument used to measure it, and it is not band-limited. This is referred to as the compensated or clean TDR response. In experiments, one can obtain an approximation to the compensated TDR response by deconvolving from the raw TDR response the derivative of the TDR response of an ideal short circuit at the end of the feed cable. The result is still band-limited, but this is the best one can do.

C. Circular Polarization

Simple coordinate transformation allows us to handle circular polarization in the frequency domain. Thus [31],

$$\begin{aligned} \tilde{h}_R &= \frac{1}{\sqrt{2}}\left(\tilde{h}_\theta - j\,\tilde{h}_\phi\right) \;, \qquad & \tilde{h}_\theta &= \frac{1}{\sqrt{2}}\left(\tilde{h}_R + \tilde{h}_L\right) \\ \tilde{h}_L &= \frac{1}{\sqrt{2}}\left(\tilde{h}_\theta + j\,\tilde{h}_\phi\right) \;, \qquad & \tilde{h}_\phi &= \frac{j}{\sqrt{2}}\left(\tilde{h}_R - \tilde{h}_L\right) \end{aligned} \,, \tag{9.4}$$

where the subscripts R and L indicate right- and left-hand circular polarization, respectively. With these forms, eqn. (6.5) becomes

$$\begin{bmatrix} \tilde{\Pi}_{rec}(\theta',\phi') \\ \tilde{Y}_{R,rad}(\theta,\phi,\theta',\phi') \\ \tilde{Y}_{L,rad}(\theta,\phi,\theta',\phi') \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h}_R(\theta',\phi') & \tilde{h}_L(\theta',\phi') \\ s\,\tilde{h}_R(\theta,\phi)/(2\pi v) & \tilde{\ell}_{RR}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{RL}(\theta,\phi,\theta',\phi') \\ s\,\tilde{h}_L(\theta,\phi)/(2\pi v) & \tilde{\ell}_{LR}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{LL}(\theta,\phi,\theta',\phi') \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\Sigma}_{R,inc}(\theta',\phi') \\ \tilde{\Sigma}_{L,inc}(\theta',\phi') \end{bmatrix} \tag{9.5}$$

While it is straightforward to formally manipulate the equations in the frequency domain, further work will be necessary to explain the physical meaning of the results for circular polarization in the time domain. We leave this for a future paper.

D. Impulse Response Example

Since we have spent so much time talking about the impulse response of an antenna, it may be helpful to show an example. In Figure 9.1 we show the impulse response and transfer function on boresight for dominant polarization of the IRA-3Q shown earlier in Figure 1.1. This was measured on the Farr Fields time domain antenna range, using a Tektronix model TDS8000B sampling oscilloscope with a model 80E04 sampling head, a Picosecond Pulse Labs model 4015C pulser, and a Farr Fields model TEM-1 TEM sensor.

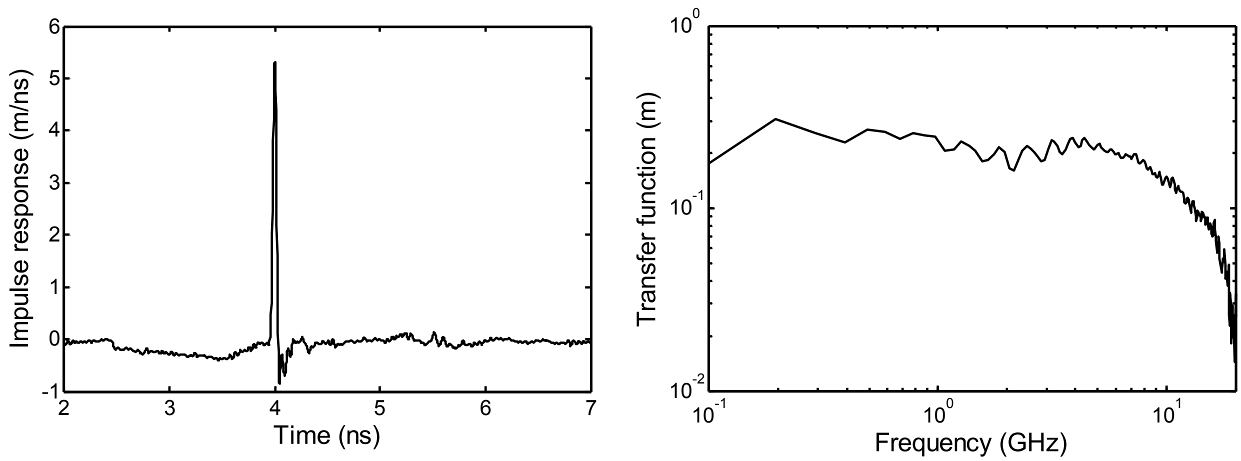

Figure 9.1. Impulse response (left) and transfer function (right) of the IRA-3Q.

E. Impulse Integral

Certain types of antennas have impulse responses on boresight that approximate a Dirac delta function over their midband range. The IRA-3Q shown earlier is one possible example, although a possibly better example is that of a long TEM horn. In these cases, the impulse response for dominant polarization on boresight may be approximated as

$$h(t) \approx h_a \, \delta(t) \; , \qquad \tilde{h} \approx h_a \; , \qquad h_a \approx \int_{\text{Impulse}} h(t)\, dt \; . \tag{9.6}$$

Here, h_a is the impulse integral, which is a scalar with units of meters. The integral is taken over the impulsive portion of the impulse response. In these cases, the antenna equations on boresight for dominant polarization, (eqns. (2.11), (2.12), (3.3), and (3.4)) are approximated in the frequency domain by

$$\begin{aligned} \frac{\tilde{E}_{rad}}{\sqrt{Z_{o2}}} &\approx \frac{s}{2\pi v} \frac{e^{-\gamma r}}{r} h_a \frac{\tilde{V}_{src}}{\sqrt{Z_{o1}}} \; , & \tilde{Y}_{rad} &\approx \frac{s}{2\pi v} h_a \; \tilde{\Pi}_{src} \\ \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} &\approx h_a \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} \; , & \tilde{\Pi}_{rec} &\approx h_a \; \tilde{\Sigma}_{\iota inc} \end{aligned} \quad , \tag{9.7}$$

where we have used both sets of notation. In the time domain these approximations become

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{Z_{o2}}} &\approx \frac{h_a}{2\pi v r} \frac{d V_{src}(t') / dt}{\sqrt{Z_{o1}}} \; , & Y_{rad}(t) &\approx \frac{h_a}{2\pi v} \Pi'_{src}(t) \\ \frac{V_{rec}(t)}{\sqrt{Z_{o1}}} &\approx h_a \frac{E_{inc}(t)}{\sqrt{Z_{o2}}} & \Pi_{rec}(t) &\approx h_a \; \Sigma_{inc}(t) \quad . \\ t' &= t - r / v \end{aligned} \tag{9.8}$$

When one calculates the impulse integral from an impulse response, there is always some ambiguity in determining the exact limits of the integration. (Note that the integral over the entire waveform must be equal to zero, because it is impossible to radiate a DC signal.) So the impulse integral can only lead to approximate results with a limited class of antennas. It may nevertheless be useful, because approximate results may be obtained without the need for a convolution.

F. Transient Antenna Pattern

A transient antenna pattern is a plot of some feature of the impulse response as a function of angle. The total transient pattern includes information from both polarizations, and a partial transient antenna pattern includes information from a single polarization. The impulse response magnitude is related to its partial components by

$$\left|\vec{h}(\theta,\phi,t)\right| = \sqrt{\left|h_\theta(\theta,\phi,t)\right|^2 + \left|h_\phi(\theta,\phi,t)\right|^2} \quad . \tag{9.9}$$

It is simplest to characterize time domain waveforms in terms of norms of various types. The total transient antenna pattern and the two partial transient antenna patterns (in θ and ϕ) are

$$\mathscr{P}_t(\theta,\phi) = \frac{\left\|\left|\vec{h}(\theta,\phi,t)\right|\right\|}{\left\|\left|\vec{h}(0,0,t)\right|\right\|} \, , \quad \mathscr{P}_\theta(\theta,\phi) = \frac{\left\|h_\theta(\theta,\phi,t)\right\|}{\left\|h_\theta(0,0,t)\right\|} \, , \quad \mathscr{P}_\phi(\theta,\phi) = \frac{\left\|h_\phi(\theta,\phi,t)\right\|}{\left\|h_\phi(0,0,t)\right\|} \, , \tag{9.10}$$

where (θ, ϕ) = (0, 0) is antenna boresight, and $\| \ \|$ is a specified norm. While these patterns are normalized to boresight, that feature is optional. Partial transient antenna patterns could also be applied to circular polarization, as described earlier in Section IX.C.

Since norms are essential to transient antenna patterns, we provide a brief review. Further information may be found in [5, 32, 33], or in any linear algebra textbook. The three criteria that must be satisfied by all norms are

$$\|f(t)\| \begin{cases} = 0 \ \text{ iff } f(t) \equiv 0 \\ > 0 \ \text{ otherwise} \end{cases} \, , \quad \|\alpha\, f(t)\| = |\alpha| \, \|f(t)\| \, , \quad \|f(t) + g(t)\| \le \|f(t)\| + \|g(t)\| . \tag{9.11}$$

The first equation states that the norm of a function can be zero if and only if the function is zero. The second equation is the linearity property, and the third equation is the triangle inequality.

A useful class of norms are the p-norms, which are defined as

$$\|f(t)\|_p = \left[\int_{-\infty}^{\infty} |f(t)|^p \, dt \right]^{1/p} \, , \qquad \|f(t)\|_\infty = \sup_t |f(t)| \quad . \tag{9.12}$$

Three p-norms are commonly of interest. The 1-norm is the area of the rectified waveform. The 2-norm is proportional to the square root of the energy in the waveform. Finally, the ∞-norm is the peak absolute magnitude of the waveform. In general, one is free to choose any norm, as long as it is clearly specified and it satisfies (9.11).

A useful variant is the derivative p-norm, or Dp-norm, where

$$\|f(t)\|_{Dp} = \|f'(t)\|_p \quad , \tag{9.13}$$

and the prime indicates a time derivative.

In some cases, one might wish to characterize the pattern in terms of the transmitting impulse response, $F(t)$, instead of the receiving impulse response, $h(t)$. This can be accomplished by applying a derivative norm to the receiving impulse response, and multiplying by $1/(2\pi v)$. (This is obvious when one recalls the self-reciprocity law, $F = h'/(2\pi v)$.) To avoid confusion, the norm should always be specified as it applies to the receiving impulse response, $h(t)$.

Once a transient antenna pattern has been established, one can consider other terms that characterize this pattern; such as transient beamwidth, $\Delta\theta_t$; and transient sidelobe level, SLL_t. In all cases, it is necessary to specify the polarization (θ, ϕ, RCP, LCP, or total magnitude), pattern cut, norm, and whether or not the pattern has been normalized to boresight. In the case of transient beamwidth, it is also necessary to specify the level at which the beamwidth is taken, for example, 3 dB below peak.

G. Bandwidth

Antenna bandwidth is [1] "the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard." It will be helpful to establish bandwidth standards for two antenna characteristics that are fundamental to power wave theory. First, reflection bandwidth, Δf_r. is the frequency range over which the reflection coefficient magnitude, $|\tilde{\Gamma}|$, is below a specified level (perhaps defaulting to –10 dB, unless specified otherwise). (This is sometimes called impedance bandwidth, although that term does not appear in [1]. However, impedance bandwidth may be confusing in antennas with waveguide feeds, in which impedance is ill-defined.) Second, transfer bandwidth, Δf_t, is the frequency range over which the magnitude of the transfer function, $|\tilde{h}|$, is above a specified level (perhaps defaulting to 3 dB below its peak value, unless specified otherwise). Dominant polarization on boresight is assumed, unless specified otherwise.

H. Group Delay

An impulse response may be separated into its magnitude and phase as

$$\tilde{h} = \left| h(\omega) \right| e^{j\psi(\omega)}, \tag{9.14}$$

where $s = j\,\omega$. From this, the group delay, t_g, is just

$$t_g(\omega) = -\frac{d\psi(\omega)}{d\,\omega} \,. \tag{9.15}$$

This is a measure of the spreading of a signal over time as a function of frequency. If an antenna has an impulse response that can be approximated by a Dirac delta function over its mid-band frequency range (as in Section IX.D), it has a group delay that is nearly flat and close to zero over a wide band of frequencies.

X. Candidate Standard Antenna Terms

We summarize here a collection of new terms that should be considered for inclusion in the antenna definitions standard [1].

- port reference impedance, Z_{o1}
- port reference impedance array in an antenna array of N elements, $Z_{o1},...,Z_{oN}$
- medium characteristic impedance, Z_{o2}, or in an antenna array of N elements, Z_{oN+1}
- velocity of propagation in the medium, v
- antenna input impedance, $\tilde{Z}_{in}$ and $Z_{in}(t)$
- antenna input impedance in an array of N elements, $\tilde{Z}_{in1}, \tilde{Z}_{in2}, ..., \tilde{Z}_{inN}$, and $Z_{in1}(t), Z_{in2}(t), ..., Z_{inN}(t)$
- source and received power waves, $\Pi_{src}(t), \tilde{\Pi}_{src}, \Pi_{rec}(t), \tilde{\Pi}_{rec}$
- incident power flux density wave, $\vec{\Sigma}_{inc}(\theta',\phi',t), \tilde{\vec{\Sigma}}_{inc}(\theta',\phi')$
- radiated radiation intensity wave, $\vec{Y}_{rad}(\theta,\phi,t), \tilde{\vec{Y}}_{rad}(\theta,\phi)$
- reflection impulse response, $\Gamma(t)$
- reflection coefficient, $\tilde{\Gamma}$
- reflection impulse response in an array of N elements, $\Gamma_1(t), \Gamma_2(t), \ldots, \Gamma_N(t)$
- reflection coefficient in an array of N elements, $\tilde{\Gamma}_1, \tilde{\Gamma}_2, ..., \tilde{\Gamma}_N$
- receiving impulse response (or simply impulse response), $\vec{h}(\theta,\phi,t)$
- receiving transfer function (or simply transfer function), $\tilde{\vec{h}}(\theta,\phi)$
- transmitting impulse response , $\vec{F}(\theta,\phi,t) = \vec{h}'(\theta,\phi,t)/(2\pi v)$
- transmitting transfer function , $\tilde{\vec{F}}(\theta,\phi) = s\,\tilde{\vec{h}}(\theta,\phi,t)/(2\pi v)$
- scattering impulse response, $\overleftrightarrow{\ell}(\theta,\phi,\theta',\phi',t)$
- scattering coefficient, $\tilde{\overleftrightarrow{\ell}}(\theta,\phi,\theta',\phi')$
- mutual coupling impulse response in an antenna array, $\Gamma_{ij}(t),\ i \neq j$
- mutual coupling coefficient in an antenna array, $\tilde{\Gamma}_{ij},\ i \neq j$
- raw TDR (Time Domain Reflectometer) response, $TDR_r(t)$
- compensated or clean TDR response, $TDR_c(t)$
- impulse integral, h_a
- total transient antenna pattern, $\mathscr{P}_t(\theta,\phi)$
- partial transient antenna pattern, $\mathscr{P}_\theta(\theta,\phi)$, $\mathscr{P}_\phi(\theta,\phi)$

- transient beamwidth, $\Delta\theta_t$
- transient sidelobe level, SLL_t
- reflection bandwidth, Δf_r,
- transfer bandwidth, Δf_t
- group delay, $t_g(\omega)$

This list is just a starting point for the discussion – undoubtedly, more terms will emerge.

We emphasize that the transmitting impulse response provides no new information, once the receiving impulse response has been specified, along with the port and medium reference impedances and the propagation velocity in the medium. For that reason, we recommend that the receiving impulse response be given the second designation of simply "impulse response."

XI. Discussion

We have identified an antenna impulse response and transfer function that fully treat all cases that are currently handled by the standard definition of gain. We summarize here the work that is left to do, and we also briefly compare our results to those of others.

The first issue that needs further investigation involves lossy media. Antenna gain, realized gain, and radar cross section are defined only for antennas embedded in lossless media, so we have limited our scope to that case. For low-loss material, one typically calculates gain for an antenna embedded in a lossless material, and adds propagation loss afterward. This approach is likely to work well in low-loss media, but it may not work well in materials with higher losses. A more rigorous solution might be to allow a complex $\tilde{Z}_{o2}$ in our formulation. Further study is needed in order to establish when this effect matters, and how best to deal with it.

A second issue worth further investigation concerns circular polarization. It is straight-forward to write the equations in the frequency domain, as we have done in Section IX.C, however, it will require further work to interpret the meaning of the results in the time domain.

A third issue that requires further investigation involves initial conditions in the source or load circuit. This might include initial voltages on a capacitor or initial currents on an inductor. Laplace transforms handle initial conditions well, so one should be able to apply to this case the usual methods developed for transient circuit theory.

Finally, we wish to briefly compare our results to earlier papers, and identify the advances that have been made. Our formulation leads to an obvious choice of antenna impulse response, $h(t)$. One can find functions proportional to $h(t)$ in most of references [4-19]. One part of the challenge lies in recognizing the scale factor that generates the simplest possible equations, and treats all cases of interest. Our approach is to normalize electric fields and voltages to the square root of the local reference impedance, and a number of authors have adopted this approach [14, 15, 34, 35]. This scaling is important, because it leads to a simple and general form of the antenna self-reciprocity law, $F = h'/(2\pi v)$. However, we treat more cases, including mismatched sources and loads, waveguide feeds, and media with arbitrary real characteristic impedance. The concept of power waves is particularly well suited to characterizing circuits containing waveguides, in which impedances are ill-defined, but reflection coefficients are available. Our approach also leads to an obvious definition of mutual coupling coefficient in antenna arrays, which is a significant advance in frequency domain antenna theory. Because we treat additional cases, our theory for the first time treats all the cases that are treated by the current definition of antenna gain. This is a critical requirement for establishing standard terminology in the time domain.

XII. Conclusions

We have introduced here the power wave theory of antennas, which addresses the problem of characterizing antennas in the time domain, using equations that are as simple as possible, eqns. (6.1) and (6.3). We have defined a number of impulse responses of antennas that, taken together, fully extend into the time domain the concepts of gain, realized gain, effective length, antenna pattern, beamwidth, radar cross section, and scattering cross section. We have defined antenna impulse response in such a way that it treats all cases that are currently treated by gain. Some further work will be useful, as is summarized in Section XI.

Our formulation also clarifies antenna performance in the frequency domain in two ways. It provides a rigorous definition of the mutual coupling coefficient in antenna arrays. It also clarifies the concept of antenna bandwidth.

A principal result of this work is that we have identified a fundamental relationship between the transmitting and receiving impulse responses, $F = h'/(2\pi v)$, the law of antenna self-reciprocity. Since this is true of all linear antennas that have no nonreciprocal components, it is unnecessary to provide both transmitting and receiving impulse responses – a single one will do. It is most convenient to establish the receiving impulse response as the default, for reasons that are explained in Section VIII.

The goal of this work was to simplify the antenna equations to the point where definitions could be agreed upon for inclusion into the antenna definitions standard [1]. Doing so would aid in characterizing the performance of antennas, especially in the time domain. In Section X we suggested about 30 new terms for inclusion in [1], and we hope they will be carefully considered.

Acknowledgments

I am grateful to Mr. Leland H. Bowen for building and testing the antenna, and processing the data. I am also grateful to Prof. J. Scott Tyo and Dr. David V. Giri, who suggested many improvements to the manuscript, and helped to clarify the terminology. Finally, a number of anonymous reviewers helped to clarify this third revision.

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