Sensor and Simulation Notes

Note 558

March 2012

Antenna Impulse Response With Arbitrary Source and Load Impedances

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Abstract

Antenna impulse response was defined in Sensor and Simulation Note 555 in terms of 50-Ω source and load impedances. While this makes the equations very simple, it is useful to extend the equations to arbitrary source and load impedances. This allows one to apply antenna impulse response to all situations. It also demonstrates that antenna impulse response, along with antenna input impedance, is sufficient to fully characterize an antenna for all sources and loads. This provides a simple way to characterize antenna performance in the time domain, analogous to the role that antenna gain serves in the frequency domain.

I. Introduction

The concept of antenna impulse response was introduced in [1] with 50-Ω source and load impedances. However, it is of interest to generalize the antenna equations to arbitrary source and load impedances, so the newly defined antenna impulse response can apply to all situations. We derive those equations here. In doing so, we confirm that the antenna impulse response, along with antenna input impedance, provides all the information that is necessary to fully describe antenna performance. Thus, antenna impulse response can serve the same role in the time domain that antenna gain does in the frequency domain.

By characterizing an antenna's performance with its impulse response, we simplify our understanding of antenna physics–especially in the time domain. For example, consider the time domain response of an Impulse Radiating Antenna (IRA), as sketched in Figure 1.1. It is common to characterize its performance differently in transmission and reception, and for different risetimes or pulse widths. On the top is the received voltage when the IRA is excited by an impulse-like electric field, with two different pulse widths. On the bottom is the radiated field when the IRA is driven by an impulse-like voltage, with two different pulse widths. Note that the bottom waveforms are proportional to the derivatives of the corresponding top waveforms. In this formulation, four waveforms are required to fully describe antenna performance. However, it was shown in [1] that a single waveform, the antenna impulse response, contains all the information in these four waveforms.

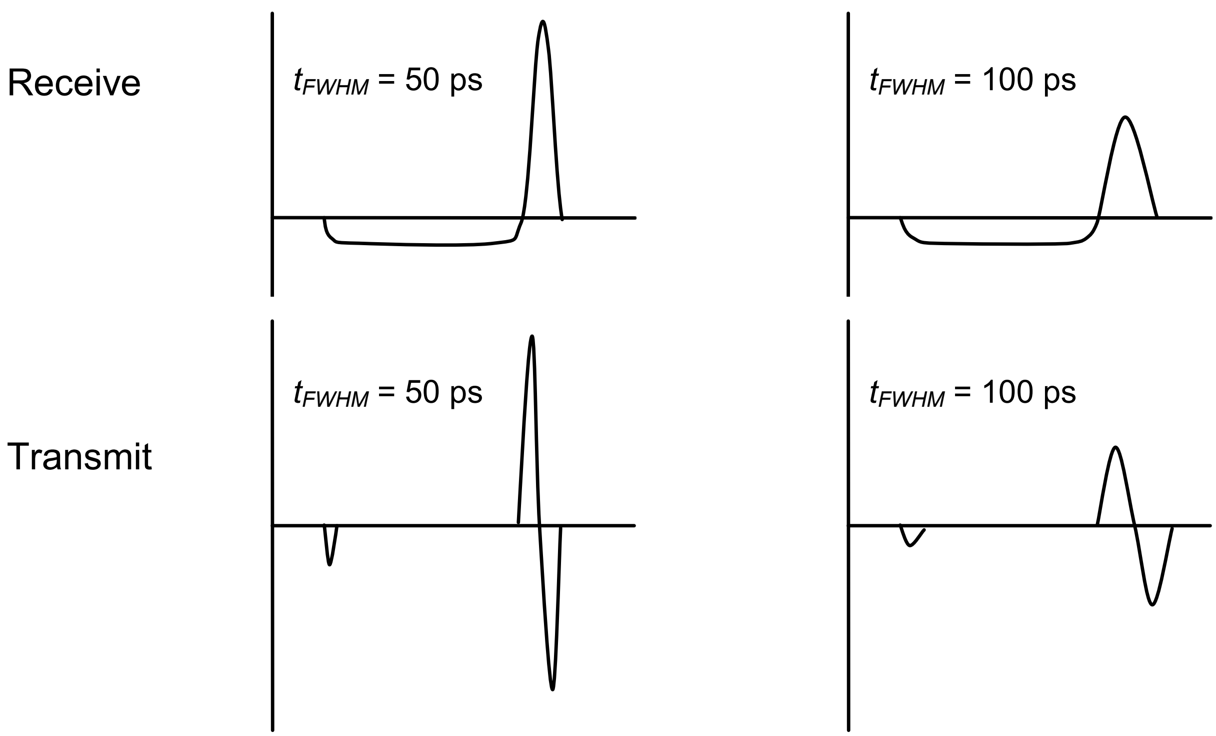

Figure 1.1. Characterizing an Impulse Radiating Antenna with four different waveforms for receive (top) and transmit (bottom), and at two different pulse widths (left and right). Note that t_{FWHM} is the Full-Width Half Max of the source voltage or incident field.

Note that all of the equations developed in this paper refer to antenna response in the dominant polarization, looking on boresight, and propagating through a lossless free space. It is straightforward to extend the equations to arbitrary polarization and look angle, and a lossy propagation constant, as shown in [1].

We begin by reviewing the relevant equations from [1]. We then extend the transmit and receive equations to arbitrary source and load impedances. We take note of the special cases of an open-circuit voltage source, a short-circuit current source, and open and short circuit loads. We express the results for all of these cases in terms of antenna impulse response.

II. Review of the Antenna Equations

Consider an antenna that transmits from a 50 Ω source, or receives into a 50 Ω load, as shown in Figure 2.1. As derived in [1, eqn. (2.1)], the antenna impulse response, $h_N(t)$, is defined by the following equations in transmission and reception,

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{377\ \Omega}} &= \frac{1}{2\pi c r}\ h_N(t) \circ \frac{d V_{src}(t')/dt}{\sqrt{50\ \Omega}}\ , \quad t' = t - r/c \\ \frac{V_{rec}(t)}{\sqrt{50\ \Omega}} &= h_N(t) \circ \frac{E_{inc}(t)}{\sqrt{377\ \Omega}} \end{aligned} \quad , \qquad (2.1)$$

where $V_{rec}(t)$ is the received voltage into a 50-Ω load or oscilloscope, and $V_{src}(t')$ is the source voltage in retarded time as measured into a 50-Ω load or oscilloscope. In a 50-Ω source, $\tilde{V}_S = 2\tilde{V}_{src}$. Furthermore, $E_{inc}(t)$ is the incident electric field, $E_{rad}(t)$ is the radiated electric field, r is the distance away from the antenna, c is the speed of light in free space, and "∘" is the convolution operator. Note also that $h_N(t)$ has units of meters per second in the time domain, and meters in the frequency domain.

Figure 2.1. Antenna transmission (left) and reception (right) for a 50-Ω source and load. Note that $\tilde{V}_S = 2\tilde{V}_{src}$ in a 50-Ω source.

The above equations are expressed alternatively in the frequency domain as

$$\begin{aligned} \frac{\tilde{E}_{rad}}{\sqrt{377\ \Omega}} &= \frac{j\omega}{2\pi c}\ \frac{e^{-jkr}}{r}\ \tilde{h}_N\ \frac{\tilde{V}_{src}}{\sqrt{50\ \Omega}} \\ \frac{\tilde{V}_{rec}}{\sqrt{50\Omega}} &= \tilde{h}_N\ \frac{\tilde{E}_{inc}}{\sqrt{377\ \Omega}} \end{aligned} \quad , \qquad (2.2)$$

where the tilde indicates a Fourier transform.

Next, we generalize the conditions in Figure 2.1 to arbitrary source and load impedances, as shown at the bottom of Figure 2.2. The special cases of open circuit voltage and short circuit currents are shown at the top and middle of the same figure.

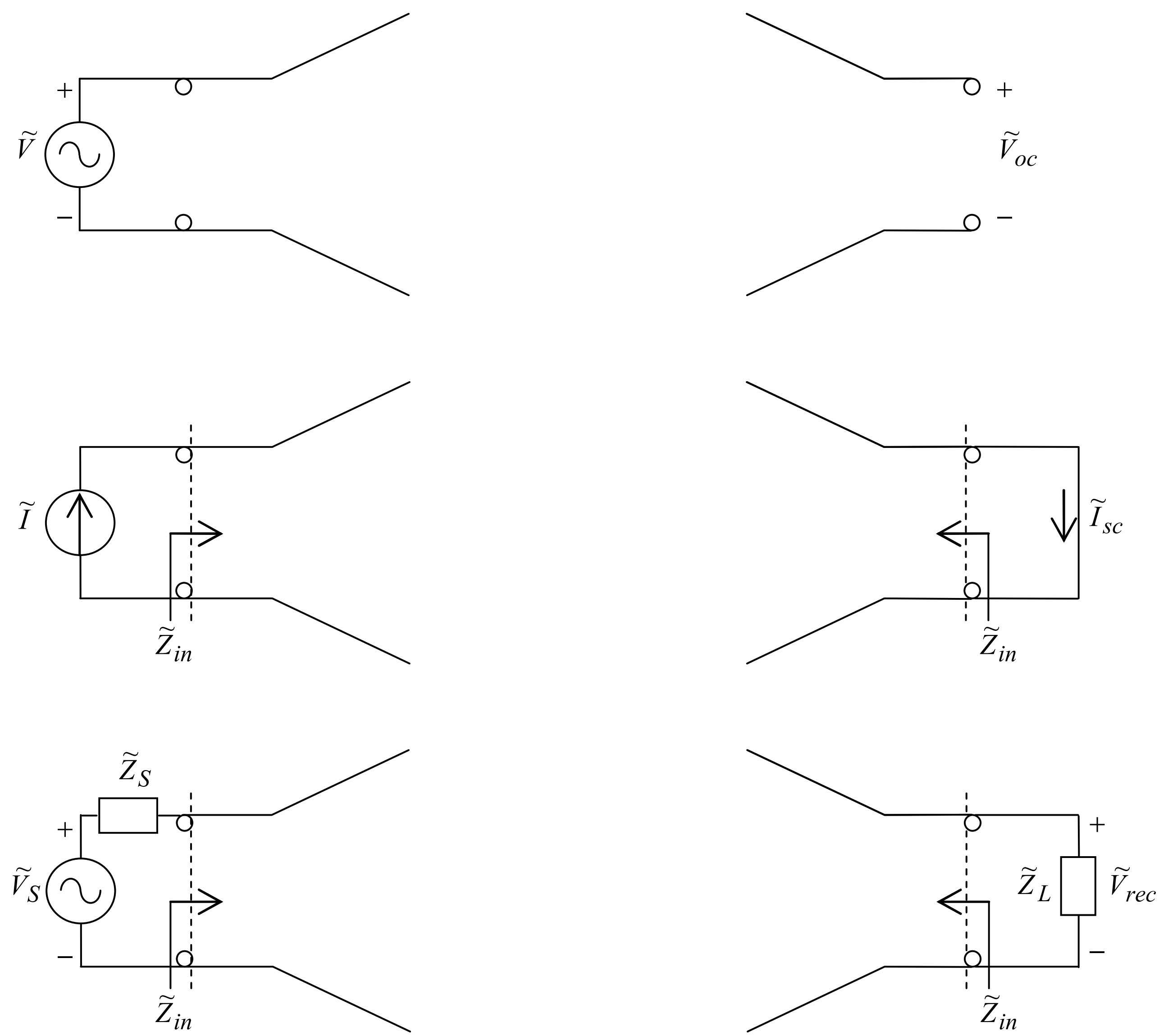

Figure 2.2. Three cases of antenna transmission (left) and reception (right) under conditions of open circuit (top), short circuit (middle), and arbitrary source and load impedances (bottom).

The equations in transmission may be expressed in one of three forms, depending on the source type; open-circuit voltage source, short-circuit current source, or loaded source [1,2]; as shown on the left in Figure 2.2. For these three cases we have

$$
\begin{aligned}
\tilde{E}_{rad} &= \frac{e^{-jkr}}{r} \tilde{F}_V \; \tilde{V} \\
&= \frac{e^{-jkr}}{r} \tilde{F}_I \; \tilde{I} \quad , \quad \tilde{F}_I = \tilde{Z}_{in} \; \tilde{F}_V \quad , \quad \tilde{I} = \frac{1}{\tilde{Z}_{in}} \tilde{V} \quad . \\
&= \frac{e^{-jkr}}{r} \tilde{F}_w \tilde{V}_S \quad , \quad \tilde{F}_w = \frac{\tilde{Z}_{in}}{\tilde{Z}_{in} + \tilde{Z}_s} \tilde{F}_V \quad , \quad \tilde{V}_S = \frac{\tilde{Z}_{in} + \tilde{Z}_s}{\tilde{Z}_{in}} \tilde{V}
\end{aligned}
\tag{2.3}
$$

Here, $\tilde{E}_{rad}$ is the radiated far field, and $k = \omega / c = 2\pi \, f / c$ is the propagation constant. For reasons of simplicity, we removed the usual dependencies on angle, polarization, and propagation through a lossy medium; all of which can easily be restored.

Next, we consider the antenna equations in reception. We consider three cases; open circuit voltage, short circuit current, and voltage across a load; as shown on the right in Figure 2.2. For these three cases we have

$$
\begin{aligned}
\tilde{V}_{oc} &= \tilde{h}_V \; \tilde{E}_{inc} \\
\tilde{I}_{sc} &= \tilde{h}_I \; \tilde{E}_{inc} \quad , \quad \tilde{h}_I = \frac{1}{\tilde{Z}_{in}} \tilde{h}_V \quad . \\
\tilde{V}_{rec} &= \tilde{h}_w \; \tilde{E}_{inc} \quad , \quad \tilde{h}_w = \frac{\tilde{Z}_L}{\tilde{Z}_{in} + \tilde{Z}_L} \tilde{h}_V
\end{aligned}
\tag{2.4}
$$

Note that we use the convention here that positive current flows into the load.

Let us now consider how one characterizes the source. In [1], the source was constrained to be a 50Ω resistor, and the transmit equation was cast in terms of $\tilde{V}_{src}$. This is the voltage seen by a 50-Ω instrument, as shown in Figure 2.3 (left). The instrument is normally a network analyzer or oscilloscope. In this case

$$\tilde{V}_{src} = \tilde{V}_{inst} = \frac{\tilde{V}_S}{2} . \tag{2.5}$$

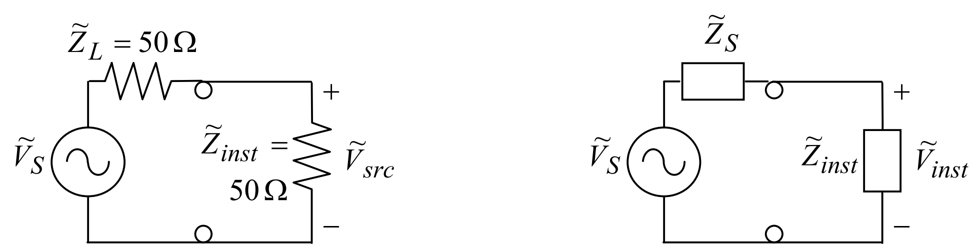

Figure 2.3. Configurations for measuring the source voltage with source and instrument impedances constrained to be 50 Ω (left), and arbitrary (right).

We now generalize to an arbitrary source impedance, as shown on the right of Figure 2.3. In this case, it is more convenient to express antenna equations in terms of $\tilde{V}_S$ instead of $\tilde{V}_{src}$. One measures the source voltage with an instrument with input impedance $\tilde{Z}_{inst}$. Now $\tilde{V}_S$ is found from

$$\tilde{V}_{inst} = \frac{\tilde{Z}_{inst}}{\tilde{Z}_S + \tilde{Z}_{inst}} \tilde{V}_S . \tag{2.6}$$

We assume that $\tilde{Z}_{inst}$` and $\tilde{Z}_s$`are known quantities, most likely from prior input impedance measurements. One reads $\tilde{V}_{inst}$ off the instrument, so the only unknown in the above equation is $\tilde{V}_S$. This is the voltage source that will be used to drive the antenna in the transmit equations to be derived later.

III. Reception into an Arbitrary Load Impedance

Let us now consider reception into an arbitrary load. From eqn. (2.4) we have

$$\tilde{V}_{rec} \;=\; \frac{\tilde{Z}_L}{\tilde{Z}_{in} + \tilde{Z}_L}\;\tilde{h}_V\;\tilde{E}_{inc}\,. \tag{3.1}$$

If we specialize the above equation to the case where $\tilde{Z}_L = 50\,\Omega$, we have

$$\tilde{V}_{rec}(\tilde{Z}_L = 50\Omega) \;=\; \frac{50\,\Omega}{\tilde{Z}_{in} + 50\,\Omega}\;\tilde{h}_V\;\tilde{E}_{inc}\,. \tag{3.2}$$

Taking the ratio of the above two equations, we find the ratio of the received voltage with arbitrary load impedance, $\tilde{Z}_L$, to that with $\tilde{Z}_L = 50\,\Omega$,

$$\frac{\tilde{V}_{rec}}{\tilde{V}_{rec}(\tilde{Z}_L = 50\Omega)} \;=\; \frac{\tilde{Z}_L}{50\,\Omega}\,\frac{\tilde{Z}_{in} + 50\,\Omega}{\tilde{Z}_{in} + \tilde{Z}_L}\,. \tag{3.3}$$

According to eqn. (2.2), the received voltage into a 50 Ω load is

$$\frac{\tilde{V}_{rec}}{\sqrt{50\,\Omega}} \;=\; \tilde{h}_N\,\frac{\tilde{E}_{inc}}{\sqrt{377\,\Omega}}\,. \tag{3.4}$$

By combining the above two equations, we find a general receive equation in terms of antenna impulse response,

$$\boxed{\frac{\tilde{V}_{rec}}{\sqrt{Z_L}} \;=\; \sqrt{\frac{\tilde{Z}_L}{50\,\Omega}}\;\frac{\tilde{Z}_{in} + 50\,\Omega}{\tilde{Z}_{in} + \tilde{Z}_L}\;\tilde{h}_N\;\frac{\tilde{E}_{inc}}{\sqrt{377\,\Omega}}}\,. \tag{3.5}$$

This is the general expression of receive voltage in terms of antenna impulse response, with arbitrary load impedance.

We now consider the two special cases of received voltage into an open circuit ($\left|\tilde{Z}_L\right| \to \infty\,\Omega$), and a short circuit ($\tilde{Z}_L = 0\ \Omega$). For these two cases, the above equation simplifies to

$$\begin{aligned} \tilde{V}_{oc} &= \tilde{h}_V\ \tilde{E}_{inc}\ , \qquad \tilde{h}_V = \frac{\tilde{Z}_{in} + 50\,\Omega}{\sqrt{50\,\Omega \times 377\,\Omega}}\ \tilde{h}_N \\ \tilde{I}_{sc} &= \tilde{h}_I\ \tilde{E}_{inc}\ , \qquad \tilde{h}_I = \frac{1}{\tilde{Z}_{in}}\,\frac{\tilde{Z}_{in} + 50\,\Omega}{\sqrt{50\,\Omega \times 377\,\Omega}}\ \tilde{h}_N \end{aligned} \quad . \tag{3.6}$$

To find $\tilde{I}_{sc}$ from eqn. (3.5), we noted that $\tilde{I}_{sc} = \tilde{V}_{rec} / \tilde{Z}_L$, and took the limit as $\left|\tilde{Z}_L\right| \to 0$.

As a check, we observe that the above expressions for $\tilde{h}_V$ and $\tilde{h}_I$ satisfy $\tilde{h}_V = \tilde{Z}_{in}\,\tilde{h}_I$, which is necessary to be consistent with eqn. (2.3).

IV. Transmission from a Source with Arbitrary Source Impedance

Next, we consider transmission from a source with arbitrary source impedance. From eqn. (2.3), we have

$$\tilde{E}_{rad} = \frac{e^{-jkr}}{r} \frac{\tilde{Z}_{in}}{\tilde{Z}_{in} + \tilde{Z}_S} \tilde{F}_V \; \tilde{V}_S . \tag{4.1}$$

Let us now specialize the above equation to the specific case where $\tilde{Z}_S = 50\,\Omega$

$$\tilde{E}_{rad}\,(\tilde{Z}_S = 50\Omega) = \frac{e^{-jkr}}{r} \frac{\tilde{Z}_{in}}{\tilde{Z}_{in} + 50\Omega} \tilde{F}_V \; \tilde{V}_S . \tag{4.2}$$

Taking the ratio of the above two equations, we find the ratio of the radiated field with arbitrary source impedance, $\tilde{Z}_S$, to that with $\tilde{Z}_S = 50\,\Omega$,

$$\frac{\tilde{E}_{rad}}{\tilde{E}_{rad}\,(Z_S = 50\Omega)} = \frac{\tilde{Z}_{in} + 50\,\Omega}{\tilde{Z}_{in} + \tilde{Z}_S} . \tag{4.3}$$

Let us now return to our expression for the field radiated from an antenna driven by a 50-Ω source, eqn. (2.2). We need to modify this to a more general form, expressing the source voltage in terms of $\tilde{V}_S$ instead of $\tilde{V}_{src}$, where the relationship between them was specified in Figure 2.3. So the field radiated from a antenna driven by a 50-Ω source, in terms of $\tilde{h}_N$ and $\tilde{V}_S$, is

$$\frac{\tilde{E}_{rad}}{\sqrt{377\,\Omega}} = \frac{j\omega}{4\pi c} \frac{e^{-j\,k\,r}}{r} \; \tilde{h}_N \; \frac{\tilde{V}_S}{\sqrt{50\,\Omega}} . \tag{4.4}$$

This is the radiated field for the case when $\tilde{Z}_S = 50\,\Omega$. To find the radiated field for arbitrary $\tilde{Z}_S$, we combine the above two equations, leading to

$$\boxed{\frac{\tilde{E}_{rad}}{\sqrt{377\,\Omega}} = \frac{j\omega}{4\pi c} \frac{e^{-jkr}}{r} \sqrt{\frac{\tilde{Z}_S}{50\Omega}} \frac{\tilde{Z}_{in} + 50\Omega}{\tilde{Z}_{in} + \tilde{Z}_S} \; \tilde{h}_N \; \frac{\tilde{V}_S}{\sqrt{\tilde{Z}_S}}} . \tag{4.5}$$

This is the general equation for radiation from a source with arbitrary feed impedance expressed in terms of the antenna impulse response.

We now consider two special cases. First, if the source is an open-circuit voltage source, then $\tilde{Z}_S = 0\ \Omega$, and $\tilde{V}_S = \tilde{V}$, so eqn. (4.5) reduces to

$$\tilde{E}_{rad} \;=\; \frac{j\omega}{4\pi c}\,\frac{e^{-jkr}}{r}\sqrt{\frac{377\ \Omega}{50\ \Omega}}\;\frac{\tilde{Z}_{in}+50\,\Omega}{\tilde{Z}_{in}}\;\tilde{h}_N\,\tilde{V}\,. \tag{4.6}$$

This may be expressed alternatively as

$$\tilde{E}_{rad} \;=\; \frac{e^{-jkr}}{r}\,\tilde{F}_V\,\tilde{V}\;, \qquad \tilde{F}_V \;=\; \frac{j\omega}{4\pi c}\sqrt{\frac{377\ \Omega}{50\ \Omega}}\;\frac{\tilde{Z}_{in}+50\,\Omega}{\tilde{Z}_{in}}\;\tilde{h}_N\,. \tag{4.7}$$

This establishes the relationship between $\tilde{F}_V$ and $\tilde{h}_N$.

Similarly, if the source is a short-circuit current source, then $\left|\tilde{Z}_S\right| \to \infty\ \Omega$, and $\tilde{V}_S / \tilde{Z}_S = \tilde{I}$, so eqn. (4.5) reduces to

$$\tilde{E}_{rad} \;=\; \frac{j\omega}{4\pi c}\,\frac{e^{-jkr}}{r}\sqrt{\frac{377\ \Omega}{50\ \Omega}}\;\left(\tilde{Z}_{in}+50\,\Omega\right)\,\tilde{h}_N\,\tilde{I}\,. \tag{4.8}$$

This may be expressed alternatively as

$$\tilde{E}_{rad} \;=\; \frac{e^{-jkr}}{r}\,\tilde{F}_I\,\tilde{I}\;, \qquad \tilde{F}_I \;=\; \frac{j\omega}{4\pi c}\sqrt{\frac{377\ \Omega}{50\ \Omega}}\;\left(\tilde{Z}_{in}+50\,\Omega\right)\tilde{h}_N\,. \tag{4.9}$$

This establishes the relationship between $\tilde{F}_I$ and $\tilde{h}_N$.

As a check, we note that the expressions for $\tilde{F}_V$ and $\tilde{F}_I$ developed in eqns. (4.7) and (4.9) satisfy $\tilde{F}_I = \tilde{Z}_{in}\,\tilde{F}_V$, which is necessary to be consistent with eqn. (2.3).

V. Discussion

Antenna impulse response may be used to describe both narrowband and broadband antennas, however, there are a few limitations on its applicability. The antenna must have a clearly defined input port. Also, the antenna may not contain a nonreciprocal material, such as ferrite.

We have defined antenna impulse response in a manner that makes the equations simplest with 50-Ω loads and sources. It should be clear, however, that we could have chosen instead to make the equations simplest when the antenna is, for example, open-circuited, or loaded by a 75-Ω resistor. We favored 50-Ω systems because that is the most common configuration in which antennas are used.

Antenna impulse response is valid only in the far field of the antenna. This may be described in the frequency domain as [3]

$$\begin{aligned} r &> 2D^2/\lambda \\ r &>> \lambda \quad ,. \\ r &>> D \end{aligned} \tag{5.1}$$

where D is the antenna diameter, and λ is the wavelength. Alternatively, D. V. Giri has formulated an analogous expression in the time domain [4]

$$r > D^2/(2ct_r)\,, \tag{5.2}$$

where t_r is the risetime of the source voltage.

To measure the complete antenna impulse response, one must use a measurement system with a sufficiently large bandwidth (or a sufficiently small risetime). One has sufficient bandwidth when increasing the system bandwidth (or reducing its risetime) no longer changes the measured antenna impulse response. When one increases the system bandwidth, it is also necessary to increase the antenna separation, consistent with eqns. (5.1) and (5.2).

VI. Conclusions

In [1], we showed that antenna impulse response, $\tilde{h}_N(\omega)$ or $h_N(t)$, fully describes antenna response with 50-Ω sources and loads. It describes an antenna in both transmission and reception, and in both the frequency and time domains. However, we had not yet addressed antenna response in the presence of arbitrary sources or loads.

In this paper, we have shown that the antenna impulse response is sufficient to fully describe antenna response with arbitrary source and load impedances. This includes the limiting cases of open-circuit voltage sources, short-circuit current sources, and open- and short-circuit loads. To fully characterize the antenna, it is also necessary to specify its input impedance.

Because only a single waveform is needed, it is not necessary to use separate waveforms to describe an antenna's performance in reception and transmission. Similarly, the antenna impulse response is independent of the waveshape parameters of the source voltage and incident field.

We showed in [1] that antenna impulse response is simply related to antenna gain, realized gain, and antenna factor. Thus, the concept of antenna impulse response extends all three of the standard descriptions of antenna performance into the time domain.

The concept of antenna impulse response makes it possible for vendors and customers to speak a common language when describing antenna performance in the time domain. This is analogous to the role that antenna gain has served for many years in the frequency domain.

We recommend that antenna impulse response be adopted by the antenna community as the standard method of describing antenna performance in the time domain. Furthermore, we recommend that IEEE adopt it as a standard term in a future version of [5].

References

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