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A Simplified Theory of Microwave Pulse Compression

Andrey D. Andreev[1], Everett G. Farr[2], and Edl Schamiloglu[1]

[1] University of New Mexico, ECE Department, Albuquerque, NM 87131
[2] Farr Research, Inc., Albuquerque, NM 87123

Abstract

This paper describes a simplified theory of the microwave pulse compressor allowing one to estimate the microwave power gain G that is possible to achieve inside a resonant cavity of a single-mode microwave pulse compressor utilizing single-arm waveguide Tee.

1. Introduction

Various experimental schemes of active HPM pulse compression have been studied during the last 25 years or so [1]. All these efforts are inspired by searching for ways to increase radiated microwave power by squeezing an initial long-duration, low-power or even CW microwave signal into a short-duration, high-power microwave pulse while keeping the total radiated microwave energy constant. The technique involves the slow excitation of a resonant cavity with a rather low-power microwave pulse under conditions when coupling between stored microwave energy and the cavity output is negligible (high Q cavity), and then firing a fast waveguide switch to destroy the cavity resonance and, in this manner, sharply increase the coupling between the stored microwave energy and the cavity output (low Q cavity). The stored microwave energy is then released during time τ that is sufficiently less than the time t required for storing the microwave energy inside the resonant cavity; the time t scales with the quality factor of a high Q cavity as $\sim Q/\omega$. Due to the fact that the stored microwave energy is released much more rapidly than it is stored in the resonant cavity, $\tau \ll t$, the output microwave power is greater than the input power by a factor of approximately t/τ.

There are basically two designs of the single-mode resonant cavity microwave pulse compressor utilizing either a single-arm waveguide Tee or a double-arm waveguide (Magic) Tee. Each design of the resonant cavity has input and output waveguides connected to the input and output arm of a Tee, respectively, and a short-circuited waveguide connected to the side arm of a Tee. The short-circuited waveguide has a switch located at the position one-quarter (or, in the worst case, some odd number of one-quarter) of the waveguide wavelength away.

At the storage mode of the microwave pulse compressor operation, the field distribution inside the resonant cavity has a minimum (where one-half of the waveguide wavelength is positioned) at the output of a Tee and a maximum (where one-quarter of the waveguide wavelength is positioned) at the switch position. When the switch is fired, the microwave pulse compressor changes from the storage to the extraction mode in such a way that the field distribution inside the waveguide suddenly changes to have a null at the switch position and maximum at the output of a Tee. In other words, when the switch is closed, the electrical length of the short-circuited waveguide is changed by one quarter of the waveguide wavelength that results in the extraction of stored microwave energy from the resonant cavity.

The present paper allows one to estimate the microwave power gain G that is possible to achieve inside the single-mode resonant cavity microwave pulse compressor utilizing a single-arm waveguide Tee.

2. Simplified theory of the microwave pulse compressor

Let us consider a simplified microwave pulse compressor (Fig. 1) consisting of: 1) an input waveguide, 2) a coupling junction (iris), and 3) a resonant cavity.

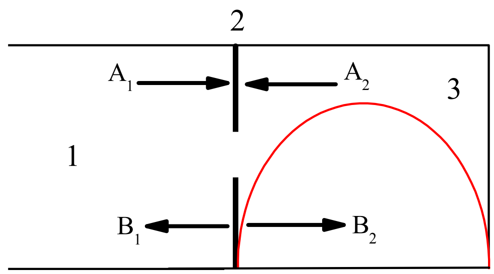

Fig. 1. Simplified diagram of the microwave pulse compressor: 1 – input waveguide, 2 – coupling junction (iris), 3 – resonant cavity with length L. A_1, A_2, B_1, B_2 are the electric field components of electromagnetic waves traveling in opposite direction relative to the diaphragm on both sides of the diaphragm.

The iris [2] may be considered as a two-port network/device with an input port facing the input waveguide and an output port facing the resonant cavity. As for any two-port network, the scattering matrix S [3] can be determined for the iris, given by

$$[S]=\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}. \tag{1}$$

If the iris is considered to be lossless, then the scattering matrix of this lossless network (1) is a unitary matrix satisfying the following condition [3, p. 178]

$$[S]^* = \{[S]^t\}^{-1}, \tag{2}$$

and by the unitary property of (2), one can write that [4, p. 204]

$$\begin{aligned} s_{11} &= s_{22} = -|s_{11}| \\ s_{12} &= s_{21} = \pm j\sqrt{1-|s_{11}|^2} \end{aligned}. \tag{3}$$

Let us determine the coupling coefficient through the iris, k, in the following way [4, p. 205]

$$|k| = \sqrt{1 - |s_{11}|^2}\,. \tag{4}$$

Equations (3) can be rewritten then as follows [4, p. 205]

$$\begin{aligned} s_{11} &= s_{22} = -\sqrt{1-k^2} \\ s_{12} &= s_{21} = jk \end{aligned}\,, \tag{5}$$

and the scattering matrix S (1) becomes [4, p. 205]

$$[S] = \begin{bmatrix} -\sqrt{1-k^2} & jk \\ jk & -\sqrt{1-k^2} \end{bmatrix}. \tag{6}$$

The electric field amplitudes of four microwave signals traveling toward the iris (A_1 and A_2) and away from the iris (B_1 and B_2) either inside (A_2 and B_2) or outside (A_1 and B_1) the resonant cavity (Fig. 1) relate to each other through the following matrix representation [3, p. 182]

$$[S] \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \tag{7}$$

Substituting (6) into (7) gives the following equation [5]

$$\begin{bmatrix} -\sqrt{1-k^2} & jk \\ jk & -\sqrt{1-k^2} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tag{8}$$

where k is the coupling coefficient (4) between the input waveguide and the resonant cavity determined by the geometry of the iris (Fig. 1). This coefficient can be low when $k \to 0$, critical, when $k = 0$, or high, when $k \to 1$.

The relation between the electric field amplitude of a microwave signal escaping the output port of the iris and entering the resonant cavity, B_2, and the electric field amplitude of a microwave signal entering the output port of the iris from the resonant cavity, A_2, can be written using the attenuation constant α, the total traveling path from B_2 to A_2, which equals to $2L$, where L is the length of the resonant cavity, and the accumulated phase shift 2φ between B_2 and A_2 over the length $2L$ [4, p. 205],

$$A_2 = -B_2 \exp(-(\alpha 2L + j2\varphi)) . \tag{9}$$

Substituting (9) into (8) gives [4, p. 205]

$$\begin{bmatrix} -\sqrt{1-k^2} & jk \\ jk & -\sqrt{1-k^2} \end{bmatrix} \begin{bmatrix} A_1 \\ -B_2 e^{-(\alpha 2L + j2\varphi)} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tag{10}$$

$$\begin{bmatrix} -\sqrt{1-k^2} \cdot A_1 + jk \cdot (-B_2 e^{-(\alpha 2L + j2\varphi)}) \\ jk \cdot A_1 + (-\sqrt{1-k^2}) \cdot (-B_2 e^{-(\alpha 2L + j2\varphi)}) \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} -\sqrt{1-k^2} A_1 - jkB_2 e^{-(\alpha 2L + j2\varphi)} \\ jkA_1 + \sqrt{1-k^2} B_2 e^{-(\alpha 2L + j2\varphi)} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{12}$$

It follows from (12) that [4, p. 205], [5, p. 372]

$$\begin{aligned} B_2 &= jkA_1 + \sqrt{1-k^2} B_2 e^{-(\alpha 2L + j2\varphi)} \\ B_2 - \sqrt{1-k^2} B_2 e^{-(\alpha 2L + j2\varphi)} &= jkA_1 \\ B_2 \left(1 - \sqrt{1-k^2} e^{-(\alpha 2L + j2\varphi)}\right) &= jkA_1 \\ B_2 &= \frac{jk}{1 - \sqrt{1-k^2} e^{-(\alpha 2L + j2\varphi)}} A_1 \end{aligned} , \tag{13}$$

and

$$\begin{aligned} B_1 &= -\sqrt{1-k^2} A_1 - jkB_2 e^{-(\alpha 2L + j2\varphi)} \\ B_1 &= -\sqrt{1-k^2} A_1 - jk \frac{jk}{1 - \sqrt{1-k^2} e^{-(\alpha 2L + j2\varphi)}} A_1 e^{-(\alpha 2L + j2\varphi)} \\ B_1 &= \left(-\sqrt{1-k^2} - jk \frac{jk}{1 - \sqrt{1-k^2} e^{-(\alpha 2L + j2\varphi)}} e^{-(\alpha 2L + j2\varphi)} \right) A_1 \\ B_1 &= -\left(\sqrt{1-k^2} - \frac{k^2 e^{-(\alpha 2L + j2\varphi)}}{1 - \sqrt{1-k^2} e^{-(\alpha 2L + j2\varphi)}} \right) A_1 \end{aligned} , \tag{14}$$

and, finally, [4, p. 205], [5, p. 372]

$$B_1 = -\left(\sqrt{1-k^2} - \frac{k^2 e^{-(\alpha 2L + j2\varphi)}}{1-\sqrt{1-k^2}\, e^{-(\alpha 2L + j2\varphi)}} \right) A_1$$
$$B_2 = \frac{jk}{1-\sqrt{1-k^2}\, e^{-(\alpha 2L + j2\varphi)}} A_1 \quad . \tag{15}$$

For a given total attenuation $\alpha 2L$ and coupling coefficient k (4), the magnitude of B_2 is maximum and $|B_1|$ assumes a minimum value when the total phase shift becomes [4, p. 205]

$$2\varphi = 2n\pi \,, \tag{16}$$

where n=0, 1, 2, 3, … . Condition (16) turns out to be the resonance condition under which B_1 and B_2 (15) become, respectively [4, p. 206]

$$B_1 = -\left(\sqrt{1-k^2} - \frac{k^2 e^{-\alpha 2L}}{1-\sqrt{1-k^2}\, e^{-\alpha 2L}} \right) A_1 = -\left(\frac{\sqrt{1-k^2} - e^{-\alpha 2L}}{1-\sqrt{1-k^2}\, e^{-\alpha 2L}} \right) A_1$$
$$B_2 = \frac{jk}{1-\sqrt{1-k^2}\, e^{-\alpha 2L}} A_1 \quad . \tag{17}$$

For a given total attenuation $\alpha 2L$, if coupling coefficient k (4) is adjustable, the maximum value of B_2 is obtained when [4, p. 206], [5, p. 373]

$$\frac{d|B_2|}{dk} = \frac{\sqrt{1-k^2} - e^{-\alpha 2L}}{\left(1-\sqrt{1-k^2}\, e^{-\alpha 2L}\right)^2 \sqrt{1-k^2}} |A_1| = 0 \,. \tag{18}$$

It follows from (18) that at optimal conditions of the microwave pulse compressor operation in the storage mode (16), the optimal coupling coefficient $k \equiv k_{opt}$ satisfies the following condition [4, p. 206], [5, p. 373]

$$\sqrt{1-k_{opt}^2} = e^{-\alpha 2L} \tag{19}$$

or

$$k_{opt} = \pm\sqrt{1-e^{-\alpha 4L}} \,. \tag{20}$$

Substituting (19) and (20) into (17) gives the electric field amplitude of a microwave signal at optimal conditions of the microwave pulse compressor operation in the storage mode (16) (resonance condition) [4, p. 206], [5, p. 373]

$$B_1 = -\left(\frac{\sqrt{1-k^2} - e^{-\alpha 2L}}{1-\sqrt{1-k^2}e^{-\alpha 2L}}\right)A_1 = -\left(\frac{e^{-\alpha 2L} - e^{-\alpha 2L}}{1-e^{-\alpha 2L}e^{-\alpha 2L}}\right)A_1 = -\left(\frac{0}{1-e^{-\alpha 4L}}\right)A_1 = 0 \tag{21}$$

$$B_2 = \frac{jk}{1-\sqrt{1-k^2}e^{-\alpha 2L}}A_1 = \frac{\pm j\sqrt{1-e^{-\alpha 4L}}}{1-e^{-\alpha 2L}e^{-\alpha 2L}}A_1 = \pm j\frac{\sqrt{1-e^{-\alpha 4L}}}{1-e^{-\alpha 4L}}A_1 = \pm j\frac{A_1}{\sqrt{1-e^{-\alpha 4L}}}$$

$$B_1 = 0 \tag{22}$$

$$B_2 = \pm j\frac{A_1}{\sqrt{1-e^{-\alpha 4L}}} = j\frac{A_1}{k_{opt}}.$$

The gain M of the electric field amplitude shows how much electric field amplitude inside the resonator B_2 greater than the electric field amplitude entering the resonator A_1 (Fig. 1). The gain M is determined, taking into account (15), by the following equation [5, p. 373]

$$M = \frac{B_2}{A_1} = \frac{jk}{1-\sqrt{1-k^2}e^{-(\alpha 2L + j2\varphi)}}. \tag{23}$$

At optimal coupling coefficient k_{opt} (19), (20), which provides maximum value of B_2, and at optimal conditions of the microwave pulse compressor operation in the storage mode (16) (resonance condition) the gain M (23) is determined, taking into account (22), by the following expression [5, p. 373]

$$M = \frac{B_2}{A_1} = \pm\frac{j}{\sqrt{1-e^{-\alpha 4L}}} = \frac{j}{k_{opt}}. \tag{24}$$

The maximum microwave power P of an electromagnetic wave within the resonant cavity under the resonance conditions (16) in relations to microwave power P_0 of an electromagnetic wave A_1 entering the resonant cavity can be written using (24) as [5, p. 373]

$$P = \left|M^2\right|A_1^2, \tag{25}$$

or

$$P = \frac{P_0}{1-e^{-\alpha 4L}}, \tag{26}$$

where $P_0=(A_1)^2$ is the input microwave power. Assuming that $|-\alpha 4L|<<1$, and $e^x \approx 1+x$ when $|x|<<1$ (Taylor's series expansion) one can reduce (15) to the following expression [5, p. 373]

$$P = \frac{P_0}{\alpha 4L}. \tag{27}$$

The amount of microwave energy W stored in a resonant cavity during the time required for the electromagnetic wave to travel within the resonant cavity from input to output, $2L$, can be written using microwave power (26), (27) and group velocity υ_g as [4, p. 420], [5, p. 373]

$$W = \frac{2L}{\upsilon_g} P, \tag{28}$$

where

$$\upsilon_g = \frac{c\lambda_0}{\lambda_g} = \frac{\omega_0 \lambda_0^2}{2\pi\lambda_g}. \tag{29}$$

The microwave energy W_1 dissipated inside the resonant cavity during one period of oscillations (per a single radian) can be written as [4, p. 421]

$$W_1 = 2\frac{(\alpha 2L)P}{\omega_0}. \tag{30}$$

Equations (28) and (30) can now be used to write the quality factor of the resonant cavity Q [4, p. 421], [5, p. 374]

$$Q = \frac{W}{W_1} = \frac{2L}{\upsilon_g} P \bigg/ 2\frac{(\alpha 2L)P}{\omega_0}, \tag{31}$$

$$Q = 2LP \frac{2\pi\lambda_g}{\omega_0 \lambda_0^2} \frac{\omega_0}{4\alpha LP} = \frac{\pi\lambda_g}{\alpha\lambda_0^2}. \tag{32}$$

The attenuation constant α can then be rewritten using (32) as [5, p. 374]

$$\alpha = \frac{\pi\lambda_g}{Q\lambda_0^2} = \frac{D}{Q}, \tag{33}$$

where $D=\pi\lambda_g/(\lambda_0)^2$ is determined as the geometrical factor of the resonant cavity.

Substituting (33) into (20) and assuming that $|-D4L/Q|<<1$, and $e^x \approx 1+x$ when $|x|<<1$ gives a relation between the optimal coupling coefficient k_{opt}, quality factor Q, and geometrical factor D of the resonant cavity at optimal conditions of the microwave pulse compressor operating in the storage mode

$$k_{opt}^2 = 4\frac{D}{Q}L. \tag{34}$$

Substituting (34) into (24)) gives a relation between the maximum gain G of the microwave power, the quality factor Q, and the geometrical factor D of the resonant cavity at optimal conditions (16) of the microwave pulse compressor operating in the storage mode

$$G = M^2 = \frac{Q}{4DL} = \frac{Q}{4L}\frac{\lambda_0^2}{\pi\lambda_g}. \tag{35}$$

Substituting (32) into (35) gives the following expression for the microwave power gain at resonance (16) for an optimal coupling coefficient k_{opt},

$$G = \frac{1}{4\alpha L}. \tag{36}$$

The microwave power gain G (35) is what one can actually be measured in an experiment by comparing input P_0 and output P_1 microwave power.

3. Concluding Remarks

These results provide an upper bound on the maximum cavity gain for an idea cavity resonator, similar to that used in pulse compression. From equation (36) we infer the maximum gain that can be realized, and the fact that gain varies inversely with cavity length. This will be useful when interpreting the results of future experiments in microwave pulse compression.

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