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Comments on the paper "A Power Wave Theory of Antennas." By Everett G. Farr; Published in FERMAT (www.e-fermat.org) / Articles / 2015 Vol7.

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Author Response:

Here I continue to respond to comments on my recent paper [1]. I address here Comment #1 [2], which was written anonymously. The examples here also address the request in Comment #2 [3] for examples.

1) Equivalent area and effective height as a function of frequency are well established concepts in Antenna theory. Your definitions of these quantities in spectral domain, appear different from the established concepts with the same names. It confuses the readers and it might be prudent to change your terminology.

The commenter has made a very serious charge. My paper would be worthless if any terminology in my paper differed from the antenna definitions standard [4]. Of course, there are no differences – my paper is everywhere consistent with [4]. The commenter offers no evidence to the contrary.

To show that my definitions are consistent with [4], I consider the two terms whose definitions the commenter accuses me of changing. First, I consider "equivalent area." This is a term that does not appear in the antenna definitions standard [1], and it does not even appear in my paper! However, I do develop "effective area" in eqn. (4.14) of [1]. After chatting with the editor, he assures me that the commenter intended that "equivalent area" and "effective area" should be interchangeable. I'll show later that this is a very bad assumption, but let's go with it for now.

From the antenna definitions standard [4] we have

> **effective area, partial (of an antenna for a given polarization and direction):** In a given direction, the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction and with a specified polarization differing from the receiving polarization of the antenna.
>
> NOTE 1— If the direction is not specified, the direction of maximum radiation intensity is implied.

NOTE 2— The effective area of an antenna in a given direction is equal to the square of the operating wavelength times its gain in that direction divided by 4π.

NOTE 3— For an active receiving antenna, available power is the active antenna available power. *See:* **active antenna, available power.**

I have underlined Note 2 for emphasis. Now look at eqn. (4.12) in my paper

$$A_e = \frac{\lambda^2}{4\pi} \widetilde{G} , \tag{4.12}$$

Compare this to Note 2 above. These look the same to me. Why does the commenter think they are different?

Next, we consider effective height. After the editor checked with the commenter, it turns out he intended effective length, not effective height. (In the antenna definitions standard, effective height and effective length have completely different meanings.) Again I quote the antenna definitions standard [4],

effective length of a linearly polarized antenna: For a linearly polarized antenna receiving a plane wave from a given direction, the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna's polarization.

Compare this to my paper, on page 8,

Effective length is the open circuit voltage in response to an incident plane wave. This is already defined in eqn. (2.2) as $\widetilde{h}_V$, …

In eqn. (2.2) I define $\widetilde{h}_V$ as

$$\widetilde{V}_{oc} = \widetilde{h}_V \, \widetilde{E}_{inc} \, . \tag{2.2}$$

Compare this to the above definition. They are the same. It's curious that the commenter thinks they are different.

2) Your paper has no example calculations of antennas. This makes me question its utility. Can you apply your equations to at least 2 cases?

Since the commenter is interested in Baum's D-dot and B-dot sensors, I consider those two cases.

A. D-dot Sensor

We begin with the D-dot sensor, which is an electrically small electric dipole driving a resistive load of Z_{o1}. The received voltage across the load in both the time and frequency domains is [5, p. 86]

$$\begin{aligned} \tilde{V}_{rec} &= A_{eq}\, Z_{o1}\, s\, \tilde{D}_{inc} = \varepsilon\, A_{eq}\, Z_{o1}\, s\, \tilde{E}_{inc} \\ V_{rec}(t) &= \varepsilon\, A_{eq}\, Z_{o1} \frac{d\, E_{inc}(t)}{dt} \end{aligned} \quad , \tag{1}$$

where ε is the permittivity of the surrounding medium. I am restricting the treatment for now to dominant polarization at an angle of maximum coupling. The general expression for antenna transfer function and impulse response are in eqns. (2.11) and (2.15) in [1],

$$\begin{aligned} \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} &= \tilde{h}\, \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} \\ \frac{V_{rec}(t)}{\sqrt{Z_{o1}}} &= h(t) * \frac{E_{inc}(t)}{\sqrt{Z_{o2}}} \end{aligned} \quad , \tag{2}$$

where " * " is the convolution operator and Z_{o2} is the impedance of the surrounding medium. This is normally free space, in which case $Z_{o2} = 120\,\pi\,\Omega$. Comparing eqns. (1) and (2), we find the transfer function and impulse response of the D-dot sensor as

$$\begin{aligned} \tilde{h} &= \varepsilon\, A_{eq} \sqrt{Z_{o1} Z_{o2}}\, s &&= \frac{A_{eq}}{v} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\, s \\ h(t) &= \varepsilon\, A_{eq} \sqrt{Z_{o1} Z_{o2}}\, \delta'(t) &&= \frac{A_{eq}}{v} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\, \delta'(t) \end{aligned} \tag{3}$$

where $\delta'(t)$ is the time derivative of the Dirac delta function, and we have used $\varepsilon = 1/(Z_{o2}\, v)$. Looking into the port, the input impedance looks like a capacitor of value C. The reflection coefficient looking into the port is therefore

$$\tilde{\Gamma} = \frac{1/(sC) - Z_{o1}}{1/(sC) + Z_{o1}} = \frac{1 - s\tau_e}{1 + s\tau_e} \quad , \qquad \tau_e = Z_{o1} C \quad . \tag{4}$$

With these parameters defined, we can now calculate all the common antenna parameters listed in Section IV of [1]. From eqn. (4.4), realized gain is

$$G_r(s) = \frac{4\pi}{\lambda^2} |\tilde{h}|^2 = \frac{4\pi}{\lambda^2} \frac{A_{eq}{}^2}{v^2} \frac{Z_{o1}}{Z_{o2}} |s|^2 \tag{5}$$

For the usual case where $s = j2\pi f$, and noting that $f = v/\lambda$, this simplifies to

$$G_r(f) = 16\pi^3 A_{eq}{}^2 \frac{Z_{o1}}{Z_{o2}} \frac{f^4}{v^4} = 16\pi^3 A_{eq}{}^2 \frac{Z_{o1}}{Z_{o2}} \frac{1}{\lambda^4} \tag{6}$$

Next, gain can be found from eqn. (4.8) in [1],

$$G(s) = \frac{G_r(s)}{1 - \left|\tilde{\Gamma}\right|^2} \tag{7}$$

The effective length is calculated from eqn. (4.11) in [1] as

$$\begin{aligned} \tilde{h}_V &= \frac{\tilde{Z}_{in} + Z_{o1}}{Z_{o1}} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\, \tilde{h} = \frac{1/(sC) + Z_{o1}}{Z_{o1}} \sqrt{\frac{Z_{o1}}{Z_{o2}}} \frac{A_{eq}}{v} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\, s \\ &= \frac{1 + s\tau_e}{s\tau_e} \frac{Z_{o1}}{Z_{o2}} \frac{A_{eq}}{v}\, s \end{aligned} \tag{8}$$

The effective area of the D-dot sensor is now calculated from eqn. (4.14) of [1] as

$$\begin{aligned} A_e(s) &= \frac{\left|\tilde{h}\right|^2}{1 - \left|\tilde{\Gamma}\right|^2} = \frac{1}{1 - \left|\tilde{\Gamma}\right|^2} \frac{A_{eq}{}^2}{v^2} \frac{Z_{o1}}{Z_{o2}} |s|^2 \\ A_e(f) &= \frac{4\pi^2 A_{eq}{}^2}{1 - \left|\tilde{\Gamma}\right|^2} \frac{Z_{o1}}{Z_{o2}} \frac{f^2}{v^2} = \frac{4\pi^2 A_{eq}{}^2}{1 - \left|\tilde{\Gamma}\right|^2} \frac{Z_{o1}}{Z_{o2}} \frac{1}{\lambda^2} \end{aligned} \;. \tag{9}$$

This completes the calculation of the various parameters of a D-dot sensor.

I hope that eqn. (9) persuades everyone about the importance of using precise terminology. Effective area, A_e, and equivalent area, A_{eq}, are two very different quantities.

To generalize the above to arbitrary angle of incidence, multiply the transfer function and impulse response by $\cos(\theta)$, where θ is the angle between the angle of incidence and the angle of maximum coupling. One would then modify the subsequent formulas accordingly.

B. B-dot Sensor

Next, we consider a B-dot sensor, or electrically small magnetic dipole driving a resistive load of Z_{o1}. The received voltage across the load in both the time and frequency domains is [5, p. 94]

$$
\begin{aligned}
\tilde{V}_{rec} &= A_{eq}\, s\, \tilde{B}_{inc} = \frac{s\, A_{eq}}{v} \tilde{E}_{inc} \\
V_{rec}(t) &= \frac{A_{eq}}{v} \frac{d\, E_{inc}(t)}{dt}
\end{aligned} \tag{10}
$$

where we have used $\tilde{B}_{inc} = \tilde{E}_{inc} / v$. As before, this treatment is for dominant polarization at an angle of maximum coupling. Compare this now to eqn. (2), and we get

$$
\begin{aligned}
\tilde{h} &= \frac{A_{eq}}{v} \sqrt{\frac{Z_{o2}}{Z_{o1}}}\, s \\
h(t) &= \frac{A_{eq}}{v} \sqrt{\frac{Z_{o2}}{Z_{o1}}}\, \delta'(t)
\end{aligned} \quad . \tag{11}
$$

Note the similarity to eqn. (3). Looking into the port, the input impedance looks like an inductor of value L. The reflection coefficient looking into the sensor from the port is

$$
\tilde{\Gamma} = \frac{sL - Z_{o1}}{sL + Z_{o1}} = \frac{s\tau_m - 1}{s\tau_m + 1} \quad , \qquad \tau_m = L / Z_{o1} \ . \tag{12}
$$

The antenna parameters are calculated as before. Thus, realized gain and gain are

$$
\begin{aligned}
G_r(s) &= \frac{4\pi}{\lambda^2} |\tilde{h}|^2 = \frac{4\pi}{\lambda^2} \frac{A_{eq}^{\ 2}}{v^2} \frac{Z_{o2}}{Z_{o1}} |s|^2 \\
G_r(f) &= 16\pi^3 A_{eq}^{\ 2} \frac{Z_{o2}}{Z_{o1}} \frac{f^4}{v^4} = 16\pi^3 A_{eq}^{\ 2} \frac{Z_{o2}}{Z_{o1}} \frac{1}{\lambda^4} \\
G(s) &= \frac{G_r(s)}{1 - |\tilde{\Gamma}|^2}
\end{aligned} \tag{13}
$$

The effective length is calculated from eqn. (4.11) in [1] as

$$
\begin{aligned}
\tilde{h}_V &= \frac{\tilde{Z}_{in} + Z_{o1}}{Z_{o1}} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\, \tilde{h} = \frac{sL + Z_{o1}}{Z_{o1}} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\, \frac{A_{eq}}{v} \sqrt{\frac{Z_{o2}}{Z_{o1}}}\, s \\
&= (s\tau_m + 1)\, \frac{A_{eq}}{v}\, s
\end{aligned}
\tag{14}
$$

The effective area of the B-dot sensor is now calculated as before from (4.14), leading to

$$
\begin{aligned}
A_e(s) &= \frac{\left|\tilde{h}\right|^2}{1 - \left|\tilde{\Gamma}\right|^2} = \frac{1}{1 - \left|\tilde{\Gamma}\right|^2} \frac{A_{eq}{}^2}{v^2} \frac{Z_{o2}}{Z_{o1}} |s|^2 \\
A_e(f) &= \frac{4\pi^2 A_{eq}{}^2}{1 - \left|\tilde{\Gamma}\right|^2} \frac{Z_{o2}}{Z_{o1}} \frac{f^2}{v^2} = \frac{4\pi^2 A_{eq}{}^2}{1 - \left|\tilde{\Gamma}\right|^2} \frac{Z_{o2}}{Z_{o1}} \frac{1}{\lambda^2}
\end{aligned}
\quad .
\tag{15}
$$

This completes the calculation of the various parameters of a B-dot sensor. Modify this for off-boresight incidence as for the D-dot sensor

3)Dr. Baum's B-dot (receiving loop antenna) and D-dot (receiving dipole antenna) electrically small loop and dipole antennas respectively have inductive and capacitive impedances which seem to create problems in your equations.

There's no problem with an antenna with inductive or capacitive input impedances, as shown in the two examples above.

References

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