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Ten Fundamental Antenna-Theory Puzzles Solved by the Antenna Equation

A remarkable array of solutions.

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Describing the performance of antennas in the time domain has long been problematic. No standard definitions exist, for example, in the *IEEE Standard Definitions of Terms for Antennas* [1]. This makes it difficult for ultrawideband (UWB) antenna manufacturers to compare their antennas to each other. This contrasts sharply with the situation in the frequency domain, where one commonly uses gain or realized gain, as defined in [1]. With no accepted standards, UWB antenna development is hindered. To address this, the antenna equation has recently been developed. However, for it to become widely adopted, more examples of its usefulness are needed.

In this article, 10 fundamental antenna-theory puzzles are solved using the antenna equation. Chief among these is how to combine gain with a meaningful phase and what the time domain analog of gain is. It is shown how to relate the antenna impulse response to realized gain. It is also explained how to use signal-flow graphs to simplify and solve complicated antenna problems. With these examples, we should finally be ready to adopt as standards new terms that emerge from the antenna equation.

INTRODUCTION

To create the needed standards, the antenna equation was introduced by this author in [2]–[5]. The antenna equation simplifies the equations that describe antennas in both the time and frequency domains. However, before new standards are established, it would be helpful to show that the antenna equation applies to a broad spectrum of problems. To that end,

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10 fundamental antenna-theory puzzles are solved here on a wide variety of subjects.

Besides showing how to describe antenna performance in the time domain, the antenna equation also shows how to characterize phase in more detail in the frequency domain. One might think that the frequency domain description of antennas has been fully developed. However, almost no mention of phase appears in the antenna definitions standard [1], and what is there is incomplete. Once phase is clarified in the frequency domain, expressions in the time domain become straightforward.

Other very fundamental problems are addressed. Because an antenna can be described in terms of (something like) scattering parameters, it can be modeled with signal-flow graphs. This makes it easy to solve a wide variety of complicated problems with the aid of Mason's rule. For example, this approach can be used to reformulate the Friis transmission equation, which normally describes scalar power flow, into a power wave expression that includes both magnitude and phase.

Various approaches have been used previously to describe antennas in the time domain [6]–[15]. However, none describes the complete antenna equation, as presented here, and none solves complicated antenna problems using signal-flow graphs. A detailed comparison of this article to previous papers is provided in the "Comparison to Previous Formulations" section.

Some of this material has already been published by this author [2]–[5], so one might wonder what is new here. Additional examples are provided for the solution of complicated problems using signal-flow graphs. The Friis transmission equation has been reformulated, as mentioned earlier. More importantly, however, this article pulls together all of the newly solved puzzles into a single article to make a complete case for new standard antenna definitions.

TEN FUNDAMENTAL ANTENNA-THEORY PUZZLES

The 10 puzzles to be solved are as follows:

1) How can one combine antenna gain with a meaningful phase?
2) What is the time domain analog of gain?
3) How can one combine radar cross section (RCS) with a meaningful phase?
4) What is the time domain analog of RCS?
5) How can one use signal-flow graphs to simplify and solve more complicated antenna problems, such as
   - an antenna driven by a source of arbitrary impedance
   - an antenna receiving into a load of arbitrary impedance
   - an antenna with a matching circuit
   - radar scattering from an antenna with an arbitrary load
   - the two-antenna problem?

   The solution to the last problem shows how to reformulate the Friis transmission equation into a power wave expression that includes both magnitude and phase.
6) How should one describe coupling into and radiation from leaky electronic equipment?
7) How should antenna bandwidth in transmission and reception be described?
8) How should antenna patterns be described in the time domain?
9) How should one describe mutual coupling in antenna arrays with waveguide feeds?
10) Which should be preferred in publications, antenna gain or realized gain?

If one wishes to establish new standard definitions, it is necessary to not only solve these puzzles but do so with the simplest possible equations. That is the claim made in this article.

THE NEED FOR IMPROVED EQUATIONS

The most commonly used parameter to describe antennas is antenna gain. This is a scalar quantity that lacks phase, so it is clearly inadequate, for example, for describing phased arrays.

When phase is needed, antennas are normally described by their effective length [1], [16]:

$$\tilde{L}_{\mathrm{eff}} = \frac{\tilde{V}_{\mathrm{oc}}}{\tilde{E}_{\mathrm{inc}}} = \frac{4\pi r e^{\gamma r}}{s\mu}\,\frac{\tilde{E}_{\mathrm{rad}}}{\tilde{I}_{\mathrm{sc}}} \tag{1}$$

where $\tilde{V}_{\mathrm{oc}}$ is the open circuit voltage, $\tilde{E}_{\mathrm{inc}}$ is the incident electric field, $\tilde{E}_{\mathrm{rad}}$ is the radiated electric field, and $\tilde{I}_{\mathrm{sc}}$ is the short circuit current. Furthermore, $s = j\omega$ is the Laplace transform variable, $\gamma = s/v$, and v is the velocity of propagation in the medium (often c, the speed of light in free space). In addition, $v = 1/\sqrt{\mu\varepsilon}$, and μ and ε are the permeability and permittivity of the surrounding medium, respectively. The tilde indicates a Laplace transform, to distinguish between frequency domain and time domain variables. Variables with tildes are generally complex, and they usually vary with frequency.

Effective length is the only term in the antenna definitions standard [1] that treats antenna phase. However, it is insufficient to describe all cases; in particular, it does not work with waveguide feeds. To use effective length, one must measure open circuit voltages and short circuit currents, which cannot be measured in waveguide feeds. The usual workaround is to attach a waveguide-to-coax adapter and characterize the antenna combined with the adapter. However, this is incomplete because other antenna parameters, such as gain, realized gain, and effective aperture, are fully defined even with waveguide feeds.

This issue may be considered an oddity or an inconvenience in the frequency domain. Phased-array antennas are built all the time using the effective length, and they work well. However, if one resolves this issue, the time domain description of antennas becomes straightforward. Furthermore, the frequency domain becomes clearer because the antenna can be described with signal-flow graphs. This makes it easier to solve more complicated problems.

THE ANTENNA EQUATION

To make the description of antennas more complete, the antenna equation was developed in [2]–[5]. It seems necessary to review it here since it will be used to solve the 10 antenna-theory puzzles.

It is simplest to consider only the dominant polarization and far fields on boresight. The general case of two polarizations and arbitrary angles of incidence and radiation is treated briefly in the "Transformation to the Time Domain and Extension to Two Polarizations" section and in more detail in [3] and [4]. The fields and waves are expressed in the frequency domain using a two-port network formulation, as shown in Figure 1, where port 2 is a radiation port. The antenna must be linear and time invariant. The inputs and outputs are

Besides showing how to describe antenna performance in the time domain, the antenna equation also shows how to characterize phase in more detail in the frequency domain.

$$\begin{aligned}\tilde{a} &= \frac{\tilde{V}_{\mathrm{inc}}}{\sqrt{Z_{o1}}} && = \text{incident power wave}\\ \tilde{b} &= \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} && = \text{received power wave}\\ \tilde{\zeta} &= \frac{\tilde{E}_{\mathrm{inc}}}{\sqrt{\eta}} && = \text{incident power flux density wave}\\ \tilde{\xi} &= \frac{r\tilde{E}_{\mathrm{rad}}}{\sqrt{\eta}}e^{\gamma r} && = \text{radiated radiation intensity wave.}\end{aligned} \tag{2}$$

Here, Z_{o1} is the real reference impedance of the input port (often $50\,\Omega$), η is the real impedance of the surrounding medium (often $120\pi\ \Omega$), and $\tilde{Z}_{\mathrm{in}}$ is the complex impedance looking into the antenna. Note that the theory is extended to waveguide feeds as described later in this section. In addition, $\tilde{V}_{\mathrm{inc}}$ is the incident voltage wave, $\tilde{V}_{\mathrm{rec}}$ is the received voltage wave, $\tilde{E}_{\mathrm{inc}}$ is the incident plane-wave electric field at the antenna, and $\tilde{E}_{\mathrm{rad}}$ is the radiated electric far field.

The reference impedances, Z_{o1} and η, are restricted here to be real. A more general approach with complex reference impedances may be possible; however, doing so would complicate the equations without realizing any obvious benefit. If the antenna is fed by a waveguide, the analog of real reference impedances is lossless reference waveguide modes.

Because of the restriction to real reference impedances and lossless reference waveguide modes, all power, power flux density, and radiation intensity waves propagate without loss and are described as "lossless." There has been some confusion about the use of the term *power wave*, which is addressed in "A Note About Terminology Regarding Power Waves."

The two inputs and two outputs to the antenna described previously are related to each other by the antenna equation,

$$\begin{bmatrix}\tilde{b}\\ \tilde{\xi}\end{bmatrix} = \begin{bmatrix}\tilde{\Gamma} & \tilde{h}\\ s\tilde{h}/(2\pi v) & \tilde{\ell}\end{bmatrix}\begin{bmatrix}\tilde{a}\\ \tilde{\zeta}\end{bmatrix}, \tag{3}$$

where $\tilde{\Gamma}$ is the reflection coefficient, $\tilde{h}$ is the antenna transfer function, and $\tilde{\ell}$ is the radar scattering coefficient. In the time domain, these become the reflection impulse, antenna impulse, and radar scattering impulse responses, respectively. The matrix, referred to as the *generalized antenna scattering matrix* (*GASM*), is a complete description of antenna performance.

The Greek symbols ζ (zeta) and ξ (xi) may be unfamiliar to some readers. To help distinguish between the two, note that ζ appears earlier in the Greek alphabet than ξ, so $\tilde{\zeta}$ is the incident wave, and $\tilde{\xi}$ is the radiated wave. This is analogous to $\tilde{a}$ and $\tilde{b}$ as the incident and received power waves, respectively.

The two off-diagonal terms in the antenna equation are the challenging ones to derive. However, these may be found by rearranging terms from other references, i.e., Baum [6] or Davis and Licul [7]. A derivation from first principles is provided in [3] and [4].

The four waves defined in (2) are related to power as

$$\begin{aligned} P_{\mathrm{inc}}(s) &= \tfrac{1}{2}|\tilde{a}|^2 \qquad & S(s) &= \tfrac{1}{2}|\tilde{\zeta}|^2\\ P_{\mathrm{rec}}(s) &= \tfrac{1}{2}|\tilde{b}|^2 \qquad & U(s) &= \tfrac{1}{2}|\tilde{\xi}|^2,\end{aligned} \tag{4}$$

where $P_{\mathrm{inc}}(s)$ is the incident power at port 1, $P_{\mathrm{rec}}(s)$ is the received power at port 1, $S(s)$ is the incident power flux density, and $U(s)$ is the radiated radiation intensity. Note that the factors of ½ indicate that all wave amplitudes are peak values rather than the root mean square.

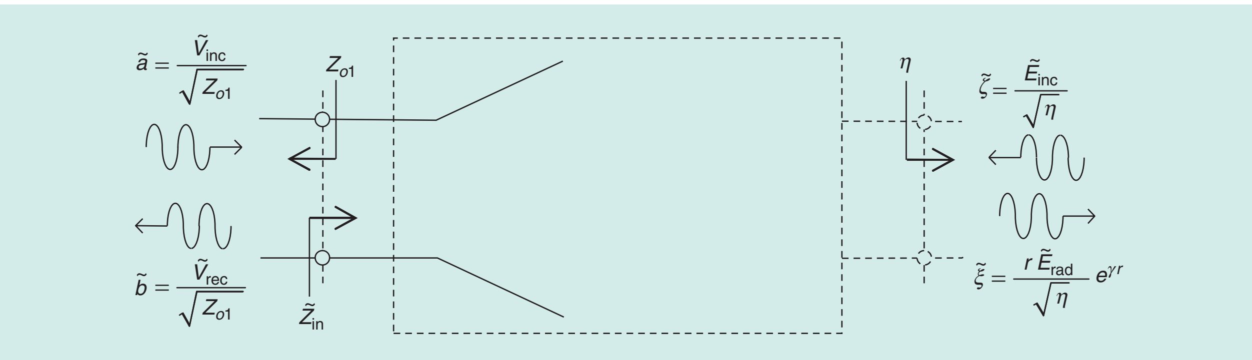

FIGURE 1. The two-port network representing an antenna, on boresight, for dominant polarization.

At this point, it is worth pausing to consider the simplicity and elegance of (3). It is impossible to imagine any simpler expression that conveys the same information. At the same time, the quantity $\tilde{h}$ plays a seminal role in expressing antenna performance, in both reception and transmission. Remarkably, it appears almost nowhere in the literature.

It is also worth pointing out the unusual nature of the GASM in (3). Scattering matrices for reciprocal devices are generally unitless and symmetric, and, if the device is lossless, the matrix is also unitary. However, in this case, none of those are true. The elements in the first column are unitless, and those of the second column have units of meters. This is necessary because the units of $\tilde{\zeta}$ differ from those of $\tilde{a}$, $\tilde{b}$, and $\tilde{\xi}$. This will seem quite strange to most readers, as it originally did to this author. However, for the case of antennas, extremely useful and simple results

With a waveguide feed, the incident and received waves are just those that would be measured by a vector network analyzer, so they are also easily measurable.

A NOTE ABOUT TERMINOLOGY REGARDING POWER WAVES

The terminology in this article has caused a great deal of confusion, and there seems to be no easy way to avoid it. There are two distinct versions of power waves in the literature, a simple version and a more complete one. The simple version is a subset of the more complete version. Confusion is caused by mistaking one for the other.

The simple version of power waves, used throughout this article, is commonly employed in introductory microwave circuit courses, although it usually is not referred to as such. In this context, power waves are the incident and scattered waves of a two-port network described by scattering parameters (S-parameters). These are incident and scattered voltage waves normalized to the square root of a real reference impedance. (With waveguide feeds, the situation is a bit more complicated, but the concept is still valid.) Such waves have units of square root of power, and they maintain full phase information. The distinguishing characteristic of these waves is that they propagate on lossless transmission lines or waveguides where they connect to the network.

The simple version of power waves goes by a variety of names in the literature. Gonzalez [17, p. 23] uses "normalized voltage wave," and Liao [S1] uses "normalized voltage." Vendelin [S2] and Neitz et al. [24] use "power wave." Pozar [18, p. 204] and Adam [S3] use just "wave." Finally, Altman [S4] uses "normalized wave." It was necessary to use "power wave" in this article to distinguish it from "power flux density wave" and "radiation intensity wave."

The more complete version of power waves has been described by Kurokawa [S5], Gonzalez [17, pp. 45–60], and Llorente-Romano et al. [S6]. The best way to describe it is to quote from the abstract by Kurokawa:

> This paper discusses the physical meaning and properties of the waves defined by
>
> $$a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\mathrm{Re}Z_i|}}, \quad b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{|\mathrm{Re}Z_i|}}, \qquad \text{(S1)}$$
>
> where V_i and I_i are the voltage at and the current flowing into the ith port of a junction, and Z_i is the impedance of the circuit connected to the ith port. The square of the magnitude of these waves is directly related to the exchangeable power of a source and the reflected power.

This more complete version reduces to the simpler version if Z_i is real.

The more complete version of power waves generates some unusual effects. In particular, the incident and scattered waves do not obey superposition and are, therefore, not separable [S7]. If superposition does not apply, then the entire theory behind the antenna equation falls apart. However, superposition is indeed satisfied in the simple version of power waves, so there is no problem.

One must then ask whether the more complete version of power waves is ever necessary for the complete description of any antenna. After all, antennas are often fed with lossy transmission lines. To find the answer, one can look to microwave circuit theory. Two-port circuits are routinely characterized with respect to, for example, a perfect 50-Ω line or a lossless WR-90 waveguide. If a lossy line is attached, that is added as a separate circuit element. Therefore, a complete description is possible even with idealized feeds and real reference impedances. That is as true of antennas as it is for circuits.

To distinguish between the two versions of power waves, the simpler version is referred to as a *lossless power wave*, since it propagates without loss. To be consistent, one then uses "lossless power flux density wave" and "lossless radiation intensity wave." Instead of adding a clumsy modifier every time a wave is mentioned, it is easier to just specify that all waves in this article are lossless, as was done here.

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are obtained when one relaxes the usual rules, as will soon become apparent.

RELATIONSHIP TO CLASSICAL PARAMETERS

The quantities that emerge from the antenna equation have simple relationships to more commonly used descriptors. For example, gain, $G(s)$, and realized gain, $G_r(s)$, depend upon the antenna transfer function, $\tilde{h}$, as

$$G_r(s) = \frac{4\pi}{\lambda^2}\frac{P_{\text{rec}}(s)}{S(s)} = \frac{4\pi}{\lambda^2}|\tilde{h}|^2 = [1-|\tilde{\Gamma}|^2]G(s), \tag{5}$$

where λ is the wavelength in the medium, and the factor in square brackets is the impedance mismatch factor. Note that $\tilde{h}$ has all of the scalar information of realized gain, and, in addition, it also has a useful phase. Thus, $\tilde{h}$ is twice as useful as realized gain since it conveys twice the information. This suggests the seminal importance of $\tilde{h}$ in antenna theory. Furthermore, when $\tilde{h}$ is transformed to the time domain, $h(t)$, it is also of seminal importance.

Note that realized gain is most meaningful when the antenna is terminated in its reference impedance. This might seem limiting since antennas are often terminated in some other impedance. However, in the field of microwave circuits, $\tilde{S}_{21}$ is, similarly, most meaningful when the circuit is loaded with reference impedances. If the antenna is terminated in an impedance other than the reference impedance, a method of calculating the received signal is provided in the "Antenna Receiving Into a Load of Arbitrary Impedance" section, using signal-flow graphs.

The effective area or effective aperture, $A_e(s)$, is related to the new parameters as

$$A_e(s) = \frac{\lambda^2}{4\pi}G(s) = \frac{|\tilde{h}|^2}{1-|\tilde{\Gamma}|^2}. \tag{6}$$

The effective length is related to the new parameters as

$$\tilde{L}_{\text{eff}} = \frac{\tilde{V}_{\text{oc}}}{\tilde{E}_{\text{inc}}} = \frac{\tilde{Z}_{\text{in}} + Z_{o1}}{\sqrt{Z_{o1}\eta}}\tilde{h}. \tag{7}$$

Note that $\tilde{L}_{\text{eff}}$ and $\tilde{h}$ are related by a complex factor since the input impedance to the antenna, $\tilde{Z}_{\text{in}}$, is, in general, complex.

Finally, the RCS of an antenna is considered. The antenna is terminated in either the antenna's reference impedance, Z_{o1}, or a nonreflecting reference waveguide load. In this condition, the RCS, $\sigma(s)$, is related to the radar scattering coefficient, $\tilde{\ell}$, by

$$\sigma(s) = 4\pi r^2\frac{|E_{\text{rad}}|^2}{|E_{\text{inc}}|^2} = 4\pi\frac{|\xi|^2}{|\zeta|^2} = 4\pi|\tilde{\ell}|^2 \tag{8}$$

In all cases, the new parameters have twice as much information as the established parameters because they include phase information. This is necessary to convert to the time domain. To illustrate the use of $\tilde{\ell}$ and $\ell(t)$, in minimum-scattering antennas, both quantities would need to be as small as possible.

It is useful at this point to consider whether the equations have really been simplified. Consider the expression of realized gain, $G_r(s)$, expressed in terms of either the effective length, $\tilde{L}_{\text{eff}}$, which is defined in [1], or the antenna transfer function, $\tilde{h}$:

$$G_r(s) = \frac{4\pi}{\lambda^2}\frac{Z_{o1}\eta}{|\tilde{Z}_{\text{in}}+Z_{o1}|^2}|\tilde{L}_{\text{eff}}|^2$$
$$G_r(s) = \frac{4\pi}{\lambda^2}|\tilde{h}|^2. \tag{9}$$

Clearly, the second expression, in terms of $\tilde{h}$, is simpler and is, therefore, preferred over the first expression with $\tilde{L}_{\text{eff}}$.

As a second example, consider the received signal into a reference load, Z_{o1}, expressed two ways:

$$\tilde{V}_{\text{rec}} = \frac{Z_{o1}}{\tilde{Z}_{\text{in}}+Z_{o1}}\tilde{L}_{\text{eff}}\tilde{E}_{\text{inc}}$$
$$\tilde{b} = \tilde{h}\tilde{\zeta}. \tag{10}$$

Once again, the second expression, in terms of $\tilde{h}$, is simpler and is, therefore, preferred over the first expression with $\tilde{L}_{\text{eff}}$. Clearly, $\tilde{h}$ is a more fundamental parameter than $\tilde{L}_{\text{eff}}$. Therefore, a clear simplification has been achieved.

One might wonder if the parameters in the antenna equation, (3), are easily measurable. They are no more difficult to measure than the electric fields or voltages one routinely measures; the only difference is that they are normalized to the square root of a real impedance. With a waveguide feed, the incident and received waves are just those that would be measured by a vector network analyzer, so they are also easily measurable. If the inputs and outputs are measurable, then the components of the matrix are easily found.

TRANSFORMATION TO THE TIME DOMAIN AND EXTENSION TO TWO POLARIZATIONS

The antenna equation, (3), is transformed to the time domain by taking its inverse Laplace transform. Thus, we have

$$\begin{bmatrix} b(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h(t) \\ h'(t)/2\pi v & \ell(t) \end{bmatrix} \stackrel{*}{\bullet} \begin{bmatrix} a(t) \\ \zeta(t) \end{bmatrix}, \tag{11}$$

where " ′ " indicates a time derivative, " $\stackrel{*}{\bullet}$ " is a matrix-product convolution operator defined as

$$\begin{bmatrix} s_{11}(t) & s_{12}(t) \\ s_{21}(t) & s_{22}(t) \end{bmatrix} \stackrel{*}{\bullet} \begin{bmatrix} a_1(t) \\ a_2(t) \end{bmatrix} = \begin{bmatrix} s_{11}(t)*a_1(t)+s_{12}(t)*a_2(t) \\ s_{21}(t)*a_1(t)+s_{22}(t)*a_2(t) \end{bmatrix} \tag{12}$$

and " $*$ " is the convolution operator.

Finally, (3) is extended to two polarizations and arbitrary angles of incidence as

$$\begin{bmatrix} \tilde{b} \\ \tilde{\xi}_\theta \\ \tilde{\xi}_\phi \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h}_\theta & \tilde{h}_\phi \\ s\tilde{h}_\theta/(2\pi v) & \tilde{\ell}_{\theta\theta} & \tilde{\ell}_{\theta\phi} \\ s\tilde{h}_\phi/(2\pi v) & \tilde{\ell}_{\phi\theta} & \tilde{\ell}_{\phi\phi} \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \tilde{\zeta}_\theta \\ \tilde{\zeta}_\phi \end{bmatrix}, \tag{13}$$

where the subscripts θ and ϕ describe the polarizations. Here, the field quantities and antenna parameters may be functions of the angles of incidence and observation. More detail may be found in [3] and [4].

PUZZLES 1 AND 2: COMBINING GAIN WITH PHASE AND THE TIME DOMAIN ANALOG OF GAIN

To begin solving the 10 puzzles, the first two are addressed. The first, how to combine gain with a meaningful phase, is clear from (5). Instead of using gain, one uses $\tilde{h}$, the antenna transfer function. Note that this description of the antenna response includes a phase that is meaningful even with waveguide feeds. The parameter currently used, effective length, fails with waveguide feeds because an open circuit voltage, in that case, is undefined. The second puzzle, the time domain analog of gain, is now simply the inverse Laplace transform of $\tilde{h}$, which is the antenna impulse response, $h(t)$. This is the parameter that should be used to compare UWB antennas to each other. Based on this argument, the parameters $\tilde{h}$ and $h(t)$ should be added to the antenna definitions standard [1].

The idea that an antenna can be represented as a signal-flow graph simplifies many problems, and five examples are illustrated here.

PUZZLES 3 AND 4: COMBINING RCS WITH PHASE AND THE TIME DOMAIN ANALOG OF RCS

The third puzzle, how to combine the RCS with a meaningful phase, is now clear from (8). One simply uses the radar scattering coefficient, $\tilde{\ell}$. The fourth puzzle, the time domain analog of RCS, is simply the inverse Laplace transform of $\tilde{\ell}$, the radar scattering impulse response, $\ell(t)$. These two parameters should also be added to the antenna definitions standard [1].

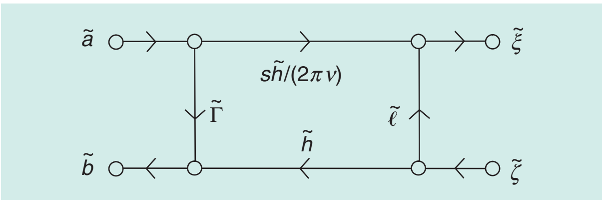

FIGURE 2. A signal-flow graph of the antenna equation, on boresight, for single polarization.

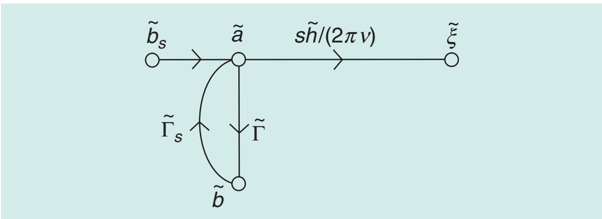

FIGURE 3. A signal-flow graph for radiation from a source of arbitrary impedance.

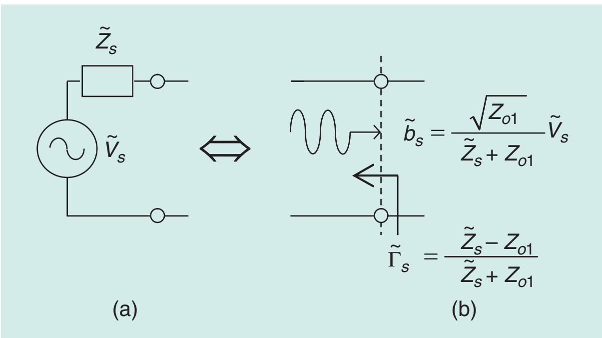

FIGURE 4. The relationship between (a) a Thévenin equivalent source and (b) a power wave source at a port with reference impedance Z_{o1}.

PUZZLE 5: SIGNAL-FLOW GRAPHS

Next, puzzle 5 asks how signal-flow graphs can be used to solve more complicated antenna problems. In microwave circuit theory, it is routine to represent circuits with signal-flow graphs and then solve them with Mason's rule [17, pp. 179–180]. Alternatively, one could use the simplification rules described, for example, by Pozar [18, pp. 214–217]. This simplifies the solution of many complex problems in microwave theory; however, this has never been extended to include antennas until now. (An earlier application of signal-flow graphs to antennas is quite different than what is shown here, as is discussed in the "Comparison to Previous Formulations" section.)

The idea that an antenna can be represented as a signal-flow graph simplifies many problems, and five examples are illustrated here. These include 1) an antenna driven by a source of arbitrary impedance, 2) an antenna receiving into a load of arbitrary impedance, 3) an antenna with a matching circuit, 4) the radar scattering of an antenna with an arbitrary load, and 5) the two-antenna problem.

The signal-flow graph of the antenna equation, (3), is shown in Figure 2. Recall that this is valid only for a single polarization. The signal-flow graph for the complete antenna equation, (13), with both field polarizations is treated in [3] and [4]. In the examples that follow, the simplified version is used because it illustrates all of the essential principles using simpler equations.

AN ANTENNA DRIVEN BY A SOURCE OF ARBITRARY IMPEDANCE

In the first example, we calculate the field radiated from an antenna driven by a source of arbitrary impedance. The signal-flow graph with a power wave source added is shown in Figure 3. One must find the ratio of $\tilde{\xi}$ to $\tilde{b}_s$, where $\tilde{b}_s = \tilde{a}$ is the power wave generated by the source. Using Mason's rule, one finds

$$\frac{\tilde{\xi}}{\tilde{b}_s} = \frac{1}{1 - \tilde{\Gamma}\tilde{\Gamma}_s}\frac{s\tilde{h}}{2\pi v}. \tag{14}$$

To find the source parameters, $\tilde{\Gamma}_s$ and $\tilde{b}_s$, one needs to find the relationship between a power wave source and a Thévenin equivalent source. That relationship is shown in Figure 4.

AN ANTENNA RECEIVING INTO A LOAD OF ARBITRARY IMPEDANCE

Next is considered the case of reception into a load of arbitrary impedance, $\tilde{Z}_\ell$. The signal-flow graph for the configuration is shown in Figure 5. It resolves as

$$\frac{\tilde{b}}{\tilde{\zeta}} = \frac{1}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell}\tilde{h}. \tag{15}$$

The new term, $\tilde{\Gamma}_\ell$, is the reflection coefficient of the load impedance, given by

$$\tilde{\Gamma}_\ell = \frac{\tilde{Z}_\ell - Z_{o1}}{\tilde{Z}_\ell + Z_{o1}}. \tag{16}$$

The port voltage and current are found as

$$\begin{aligned}\tilde{V}_1 &= Z_{o1}^{1/2}\,[\tilde{a}+\tilde{b}]\\ \tilde{I}_1 &= Z_{o1}^{-1/2}[\tilde{a}-\tilde{b}].\end{aligned} \tag{17}$$

Since $\tilde{a} = \tilde{\Gamma}_\ell \tilde{b}$, we have

$$\begin{aligned}\tilde{V}_1 &= Z_{o1}^{1/2}(1+\tilde{\Gamma}_\ell)\tilde{b}\\ \tilde{I}_1 &= Z_{o1}^{-1/2}(1-\tilde{\Gamma}_\ell)\tilde{b}.\end{aligned} \tag{18}$$

AN ANTENNA WITH A MATCHING CIRCUIT

Next, a signal-flow graph is used to calculate the characteristics of an antenna when combined with a matching circuit. The signal-flow graph of the combined antenna and matching circuit is shown in Figure 6. Using Mason's rule, the graph resolves as

$$\begin{bmatrix}\tilde{b}\\ \tilde{\xi}\end{bmatrix} = \begin{bmatrix}\tilde{S}_{m11} + \dfrac{\tilde{S}_{m21}\tilde{S}_{m12}\tilde{\Gamma}}{\Delta} & \dfrac{\tilde{S}_{m12}\tilde{h}}{\Delta}\\ \dfrac{s\tilde{S}_{m21}\tilde{h}}{2\pi v\Delta} & \tilde{\ell} + \dfrac{s\tilde{S}_{m22}\tilde{h}^2}{2\pi v\Delta}\end{bmatrix}\begin{bmatrix}\tilde{a}\\ \tilde{\zeta}\end{bmatrix}$$
$$\Delta = 1-\tilde{S}_{m22}\tilde{\Gamma}. \tag{19}$$

RADAR SCATTERING FROM AN ANTENNA WITH AN ARBITRARY LOAD

Next, a signal-flow graph is used to model the radar scattering from an antenna with an arbitrary load impedance. The signal-flow graph is shown in Figure 7. Using Mason's rule, the graph resolves as

$$\frac{\tilde{\xi}}{\tilde{\zeta}} = \tilde{\ell} + \frac{\tilde{\Gamma}_\ell}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell}\frac{s\tilde{h}^2}{2\pi v}. \tag{20}$$

Using (8), this converts to an RCS of

$$\sigma(s) = 4\pi\left|\frac{\tilde{\xi}}{\tilde{\zeta}}\right|^2 = 4\pi\left|\tilde{\ell} + \frac{\tilde{\Gamma}_\ell}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell}\frac{s\tilde{h}^2}{2\pi v}\right|^2. \tag{21}$$

Note that, if $\tilde{\Gamma}_\ell = 0$, the RCS reduces to the simpler form of (8), as it must.

THE TWO-ANTENNA PROBLEM

Finally, signal-flow graphs are used to model the two-antenna problem. This problem includes two antennas plus a propagation factor, $\tilde{\zeta}_2/\tilde{\xi}_1 = \tilde{\zeta}_1/\tilde{\xi}_2 = e^{-\gamma r}/r$ as shown in Figure 8. Using Mason's rule, the graph resolves as

$$\begin{aligned}\frac{\tilde{b}_1}{\tilde{a}_1} &= \tilde{\Gamma}_1 + \frac{e^{-2\gamma r}}{2\pi v r^2}\frac{s\tilde{h}_1^{\,2}\tilde{\ell}_2}{1-\dfrac{e^{-2\gamma r}\tilde{\ell}_1\tilde{\ell}_2}{r^2}}\\ \frac{\tilde{b}_2}{\tilde{a}_2} &= \tilde{\Gamma}_2 + \frac{e^{-2\gamma r}}{2\pi v r^2}\frac{s\tilde{h}_2^{\,2}\tilde{\ell}_1}{1-\dfrac{e^{-2\gamma r}\tilde{\ell}_1\tilde{\ell}_2}{r^2}}\\ \frac{\tilde{b}_2}{\tilde{a}_1} &= \frac{\tilde{b}_1}{\tilde{a}_2} = \frac{e^{-\gamma r}}{2\pi v r}\frac{s\tilde{h}_1\tilde{h}_2}{1-\dfrac{e^{-2\gamma r}\tilde{\ell}_1\tilde{\ell}_2}{r^2}}.\end{aligned} \tag{22}$$

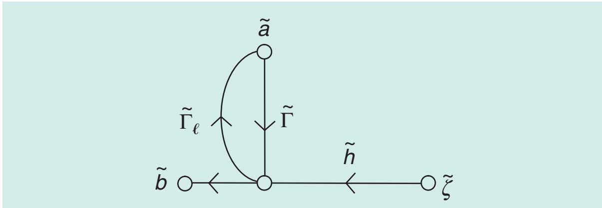

FIGURE 5. A signal-flow graph for receiving into an arbitrary load.

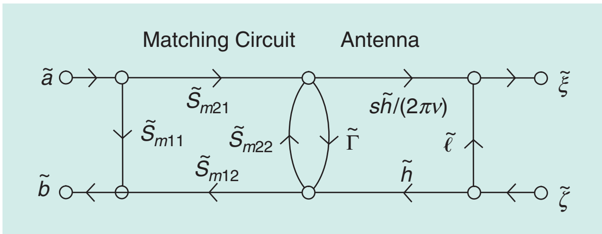

FIGURE 6. An antenna with a matching circuit.

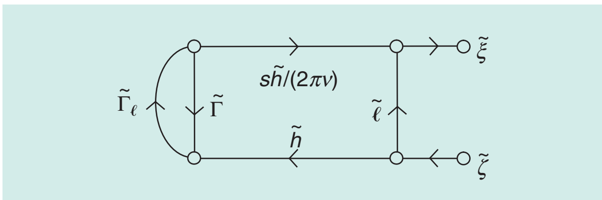

FIGURE 7. The radar scattering from an antenna with an arbitrary load.

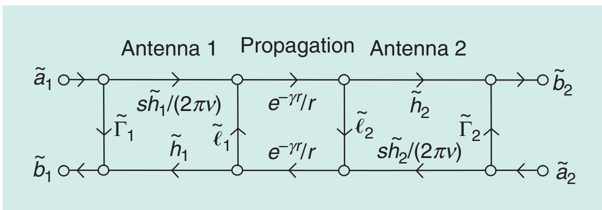

FIGURE 8. Two antennas with a propagation term.

The last equation may be considered a statement of reciprocity since $\tilde{b}_2/\tilde{a}_1 = \tilde{b}_1/\tilde{a}_2$. This is the most elegant expression of antenna reciprocity this author has seen.

In many cases, higher-order terms with $1/r^2$ dependence can be ignored in the far field. (Any calculation involving RCS is an obvious exception.) In that case, (22) simplifies to a power wave expression analogous to the Friis transmission equation [19],

$$\frac{\tilde{b}_2}{\tilde{a}_1} = \frac{\tilde{b}_1}{\tilde{a}_2} = \frac{se^{-\gamma r}\tilde{h}_1\tilde{h}_2}{2\pi v r} = \frac{je^{-\gamma r}\tilde{h}_1\tilde{h}_2}{\lambda r}$$
$$\frac{\tilde{b}_1}{\tilde{a}_1} = \tilde{\Gamma}_1, \frac{\tilde{b}_2}{\tilde{a}_2} = \tilde{\Gamma}_2, \qquad (23)$$

where we have used $s/(2\pi v) = j/\lambda$. This can be modeled by the signal-flow graph of Figure 9.

It is now instructive to manipulate (23) into the Friis transmission equation. To do so, we first note that

$$\frac{P_{\text{rec}}(s)}{P_{\text{inc}}(s)} = \left|\frac{\tilde{b}_2}{\tilde{a}_1}\right|^2 = \frac{|\tilde{h}_1|^2|\tilde{h}_2|^2}{\lambda^2 r^2}$$
$$P_1(s) = P_{\text{inc}}\left[1 - |\tilde{\Gamma}_1|^2\right]$$
$$P_2(s) = \frac{P_{\text{rec}}}{[1 - |\tilde{\Gamma}_2|^2]}, \qquad (24)$$

where $P_1(s)$ is the power accepted into the transmitting antenna, $P_2(s)$ is the available power at the receiving antenna, and $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ are the reflection coefficients looking into the transmitting and receiving antennas, respectively. Combining (24) with (5) and (6), one obtains the Friis transmission equation in various forms,

$$\frac{P_2(s)}{P_1(s)} = \frac{1}{\lambda^2 r^2}\frac{|\tilde{h}_1|^2}{[1 - |\tilde{\Gamma}_1|^2]}\frac{|\tilde{h}_2|^2}{[1 - |\tilde{\Gamma}_2|^2]}$$
$$= \frac{A_{e1}(s)\,A_{e2}(s)}{\lambda^2 r^2} = \left(\frac{\lambda}{4\pi r}\right)^2 G_1(s)G_2(s), \qquad (25)$$

where $\tilde{h}_1$ and $\tilde{h}_2$ are the antenna transfer functions of the transmitting and receiving antennas, respectively, $A_{e1}(s)$ and $A_{e2}(s)$ are the effective areas, and $G_1(s)$ and $G_2(s)$ are the gains. The advantage of (23) over (25) is that the former preserves phase information.

As an example, one could use two electrically small electric or magnetic dipoles, which have an antenna transfer function in the form of $\tilde{h} = Ks$, where K is a real constant provided by the manufacturer. This is a derivative sensor in the time domain. More detail is provided in [5].

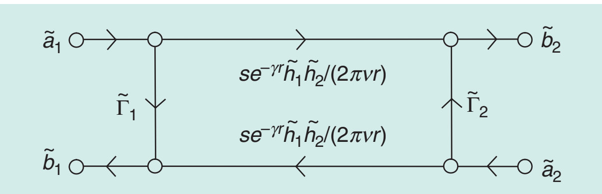

FIGURE 9. The simplified two-antenna problem.

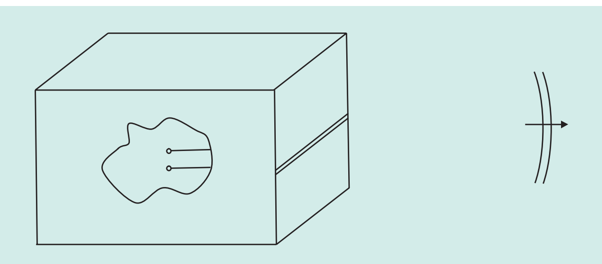

FIGURE 10. A leaky electronics cabinet with an interior port.

PUZZLE 6: LEAKAGE FROM ELECTRONIC EQUIPMENT

Puzzle 6 asks the question of how to describe coupling into and radiation from leaky electronic equipment. This is a fundamental problem in the field of electromagnetic compatibility. Consider the problem shown in Figure 10, which shows a port located inside an imperfectly shielded enclosure. It is necessary to describe both the received signal at the port when illuminated from the exterior and the radiated field when driven at the port. To be general, this must handle waveguide ports, and it must work in both the time and frequency domains.

Until recently, it was never clear how to satisfy all of the requirements. Typically, one uses open circuit voltages and short circuit currents to describe the coupling, but these fail with waveguide feeds. On the other hand, if one treats this system as an unintentional antenna, then the antenna equation provides the answer. With that understanding, all of the parameters of the antenna equation, including $\tilde{h}$ and $h(t)$, provide the most concise and complete description.

PUZZLE 7: BANDWIDTH

Puzzle 7 asks the question of how to describe the antenna bandwidth in transmission and reception. The usual method of describing the antenna bandwidth looks only at the reflection coefficient at the antenna port. This parameter is sometimes called the *impedance bandwidth*, and it is typically defined as the frequency range over which the reflection coefficient magnitude, $|\tilde{\Gamma}|$, is below a specified level (typically –10 dB). While this has the advantage of being easy to measure, it says little about the signal that is transmitted or received.

A second parameter is needed, which is harder to measure but seems more important. This is called the *transfer bandwidth*, and it is the frequency range over which the magnitude of the antenna transfer function, $|\tilde{h}|$, is above a specified level (perhaps defaulting to 3 dB below its peak value). The seminal nature of $\tilde{h}$ in the antenna equation makes this choice obvious. Both bandwidths are useful and should be specified; they should be added to the antenna definitions standard [1] since neither is currently in it.

PUZZLE 8: TRANSIENT ANTENNA PATTERNS

Puzzle 8 asks the question of how to describe antenna patterns in the time domain. Currently, no definition for *transient antenna pattern* is in our antenna definitions standard; however, it would be quite useful.

A transient antenna pattern is a plot of some feature of the antenna transient response as a function of angle. It seems preferable that the transient pattern be as meaningful in transmission as in reception since that is true of other antenna parameters, such as gain, realized gain, and effective aperture. It also would seem useful to isolate the antenna characteristics from the source characteristics. (An example of where they are combined is given in [20].) To realize this, one can plot some characteristic of the antenna impulse response, $h(t)$, as a function of angle. The seminal importance of $h(t)$ makes this choice obvious.

Puzzle 6 asks the question of how to describe coupling into and radiation from leaky electronic equipment. This is a fundamental problem in the field of electromagnetic compatibility.

When describing transient antenna patterns, one must first specify whether one is interested in the total or partial pattern. The total transient pattern includes information from both polarizations, and a partial transient antenna pattern includes information from a single polarization. The total impulse response magnitude is related to its partial components by

$$|\vec{h}(\theta,\varphi,t)| = \sqrt{|h_\theta(\theta,\varphi,t)|^2 + |h_\varphi(\theta,\varphi,t)|^2}. \qquad (26)$$

It is simplest to characterize time domain waveforms in terms of the norms of various types, as a function of angle. A norm is a measure of the "size" of a waveform according to some criterion of interest. The total and two partial transient antenna patterns (in θ and ϕ) are

$$\begin{aligned} P_t(\theta,\varphi) &= \| |\vec{h}(\theta,\varphi,t)| \| \\ P_\theta(\theta,\varphi) &= \| h_\theta(\theta,\varphi,t) \| \\ P_\varphi(\theta,\varphi) &= \| h_\varphi(\theta,\varphi,t) \|, \end{aligned} \qquad (27)$$

where "$\|\quad\|$" is a specified norm. These patterns could optionally be normalized to boresight, which has the nice feature of making them unitless.

Since norms are fundamental to the theory, it is helpful to provide a brief review of them. The three criteria that any norm must satisfy are

$$\begin{aligned} \|f(t)\| &\begin{cases} = 0 & \text{iff } f(t) \equiv 0 \\ > 0 & \text{otherwise} \end{cases} \\ \|\alpha f(t)\| &= |\alpha| \, \|f(t)\| \\ \|f(t) + g(t)\| &\le \|f(t)\| + \|g(t)\|. \end{aligned} \qquad (28)$$

The first equation states that the norm of a function can be zero if and only if the function is zero. The second equation is the linearity property, and the third equation is the triangle inequality.

A useful class of norms are the p-norms, defined as

$$\begin{aligned} \|f(t)\|_p &= \left[\int_{-\infty}^{\infty} |f(t)|^p \, dt \right]^{1/p} \\ \|f(t)\|_\infty &= \sup_t |f(t)|. \end{aligned} \qquad (29)$$

Three p-norms are commonly of interest. The 1-norm is the area of the rectified waveform. The 2-norm is proportional to the square root of the energy in the waveform. Finally, the ∞-norm is the peak absolute magnitude of the waveform. In general, one is free to choose any norm if it is clearly specified and satisfies the three conditions in (28).

PUZZLE 9: MUTUAL COUPLING IN ANTENNA ARRAYS

Next, we consider how to describe mutual coupling in antenna arrays with waveguide feeds. First, the problem with the current formulation is described, and then an improvement is proposed.

In our antenna definitions standard [1], *mutual impedance* is defined as follows:

> The mutual impedance between any two terminal pairs in a multielement array antenna is equal to the open circuit voltage produced at the first terminal pair divided by the current supplied to the second when all other terminal pairs are open-circuited.

This does not work with waveguide feeds because one cannot achieve pure open circuits, nor can one apply a short circuit current.

A better approach is to extend the full antenna equation to multiple elements in an antenna array of N elements. The antenna transfer function becomes a $2 \times N$ matrix (2 rows and N columns), $\tilde{h}_{pn}$, with $p = 1, 2$ and $n = 1, \ldots, N$, where p indicates one of two polarizations, and n represents the port number. Furthermore, the incident and received power waves become N-element vectors, $\tilde{\tilde{a}}$ and $\tilde{\tilde{b}}$, and the input reflection coefficient becomes an $N \times N$ matrix, $\tilde{\tilde{\Gamma}}$. In this formulation, $\tilde{\Gamma}_{mn}$ represents the mutual coupling coefficient seen at port m from a source at port n. Under these conditions, the antenna equation becomes

$$\begin{bmatrix} \tilde{\tilde{b}} \\ \tilde{\tilde{\xi}} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{\Gamma}} & \tilde{\tilde{h}}^{\mathrm{T}} \\ s\tilde{\tilde{h}}/(2\pi v) & \tilde{\tilde{\ell}} \end{bmatrix} \begin{bmatrix} \tilde{\tilde{a}} \\ \tilde{\tilde{\zeta}} \end{bmatrix}, \qquad (30)$$

where the superscript "T" indicates a transposed matrix. The matrix dimensions can be visualized as

$$\begin{bmatrix} [\quad]_{N\times 1} \\ [\quad]_{2\times 1} \end{bmatrix} = \begin{bmatrix} [\qquad]_{N\times N} & [\quad]_{N\times 2} \\ [\qquad]_{2\times N} & [\quad]_{2\times 2} \end{bmatrix} \begin{bmatrix} [\quad]_{N\times 1} \\ [\quad]_{2\times 1} \end{bmatrix}. \qquad (31)$$

One measures $\tilde{\Gamma}_{mn}$, the mutual coupling coefficients, by measuring conventional scattering parameters between each pair of ports. For this measurement, all unused ports are terminated with the reference impedance for that port or with a nonreflecting load in waveguide feeds. In this scenario, the array must be radiating into free space.

In the time domain, $\Gamma_{mn}(t)$ is referred to as the *mutual coupling impulse response*. When $m = n$, this is the reflection impulse response at the nth port, with all other ports terminated in a matched load. With this formulation, the definitions of the mutual coupling coefficient, $\tilde{\Gamma}_{mn}$, and mutual coupling impulse response, $\Gamma_{mn}(t)$, should be added to [1].

The same formalism may be used with multimode waveguide feeds. In this case, the antenna transfer function is $\tilde{h}_{pn}$, with $p = 1, 2$ and $n = 1, \ldots, N$, where p still refers to one of two polarizations, but n now represents the mode number.

PUZZLE 10: PUBLISH GAIN OR REALIZED GAIN?

Finally, puzzle 10 asks the question of whether it is more important to publish gain or realized gain. Due to space limitations, it is unusual to see both parameters published in an article, so one might reasonably ask which is more important.

One should prefer the parameter that most closely follows the behavior of the fundamental parameters of the antenna as revealed by the antenna equation. By now, it should be clear that the most fundamental parameter of any antenna is the antenna transfer function, $\tilde{h}$. Thus, we consider its relationship to both gain, $G(s)$, and realized gain, $G_r(s)$:

$$G_r(s) = \frac{4\pi}{\lambda^2}|\tilde{h}|^2 \qquad G(s) = \frac{1}{1-|\Gamma|^2}\frac{4\pi}{\lambda^2}|\tilde{h}|^2. \tag{32}$$

Clearly, realized gain has the simplest relationship to $\tilde{h}$. On a log scale, it would be the same shape, except for a 20-dB/decade linear factor. Antenna gain, on the other hand, is a mix of the two fundamental parameters, $\tilde{h}$ and $\tilde{\Gamma}$. Therefore, realized gain should be preferred in publications. Of course, it is also necessary to publish $\tilde{\Gamma}$ as well.

A second way of thinking about this is to compare an antenna to a two-port network in microwave circuit theory. The antenna equation makes the analogy to microwave circuit theory clear. In a microwave circuit, the quantity $\tilde{S}_{21}$ conveys great meaning. However, the quantity $\tilde{S}_{21}/\sqrt{1-|\tilde{S}_{11}|^2}$ is less meaningful and seldom calculated. The former is an analog of the square root of the realized gain, and the latter is an analog of the square root of the gain. Once again, realized gain would seem to be preferred.

There has been considerable resistance to this idea. Many have used gain for a number of years and find it difficult to change. However, this is the only mathematical justification this author has seen for choosing one over the other.

COMPARISON TO PREVIOUS FORMULATIONS

Some readers find it hard to believe that there is a new idea in the antenna equation. Formulas like these have been around for years. To prove that there is a new idea here, an extensive comparison to the literature is provided.

The new ideas presented here consist of four main parts. The first is the antenna equation itself, (3), which is a simpler and more complete expression than any of its predecessors. The second is the relationship of $\tilde{h}$ to realized gain, (5). This relationship proves that $\tilde{h}$ and its inverse transform, $h(t)$, are seminal to antenna theory. The third is the idea that the antenna equation can be modeled as a signal-flow graph to solve complicated problems. The fourth is the large number of related problems that can be addressed by the antenna equation. Taken together, these four concepts make a powerful case for including (at least) $\tilde{h}$ and $h(t)$ in the next version of the antenna definitions standard [1].

Let us now consider papers by other authors who have addressed portions of the problem. Several authors have developed expressions like the off-diagonal parts of the antenna equation without showing the on-diagonal elements. These include Davis and Licul [7], Lamensdorf and Susman [8], Shlivinski et al. [9], Kunish and Pamp [10], Smith [11], Sörgel and Wiesbeck [12], McLean et al. [13], Wiesbeck et al. [14], and Schantz [15]. While these works solve part of the problem, none uses the complete GASM as defined in (3). The complete matrix is critical for building a signal-flow graph.

Let us consider now whether any article has developed an expression analogous to (5), the relationship between $\tilde{h}$ and realized gain. This relationship is critical to establishing the seminal importance of $\tilde{h}$ and $h(t)$. The paper by Kunisch and Pamp [10] comes closest; however, they use a different normalization factor. In any case, Kunisch and Pamp do not provide a full version of the antenna equation, nor do they show the use of signal-flow graphs.

One work to which this theory is sometimes compared is that of Kerns [21]. While Kerns did not treat the time domain, his work bears a certain similarity since it is formulated in terms of waves. In one respect, Kerns' work is more complete because it is valid in the near field, where it sums a spatial spectrum of plane waves. The antenna equation described here treats only the far field. However, Kerns' formulation lacks the simplicity of (3) and (5) of this article. Consider Kerns' expression for realized gain [21, eq. (1.6-6)]:

$$G_{rq}(\mathbf{K}) = \frac{4\pi Y_0 \gamma^2 |\mathbf{s}_{q0}(\mathbf{K})|^2}{\eta_o}, \tag{33}$$

where $\mathbf{K}$ is the vector propagation constant of a spectrum of plane waves. (Note that an impedance mismatch factor was removed from the original to convert gain to realized gain.) The information in (5) is also contained in (33), but (5) is simpler and is, therefore, more suitable for establishing a standard. Furthermore, the expression in (5) suggests that the inverse Laplace transform of $\tilde{h}$, $h(t)$, will be of seminal importance. It is less clear, at least to this author, how to transform a part of the expression in (33) to find something of seminal importance in the time domain.

Another work that seems to cover similar ground is edited by Hansen [22]. He also expresses all quantities in terms of waves, but instead of a plane-wave expansion, he uses spherical waves. In [22, eq. (2.57)], he shows what he calls an *antenna scattering matrix*, which relates the incident and received modes at the antenna port to the incident and scattered spherical modes. This is valid in both the near and far fields, so it covers more cases than the antenna equation, (3), described here. However, it is a completely different expression. His scattering matrix is both unitary and unitless, whereas the GASM shown in (3) is neither. While Hansen does treat signal-flow graphs, he models a different equation than that described here.

Furthermore, Hansen's expression for realized gain is [22, eq. (2.211)]:

$$G_r(\theta,\phi) = \left| \sum_{smn} T_{smn}\, \vec{K}_{smn}(\theta,\phi) \right|^2 = |\vec{K}(\theta,\phi)|^2. \qquad (34)$$

Again, the information of (5) is contained in (34). However, (5) is simpler and offers better insight into the relationship between the realized gain and antenna impulse response.

The earliest approach to fitting scattering parameters to an antenna is probably that by Montgomery et al. [23]. The approach looks much like that of Hansen since the fields are expanded in a series of spherical waves. In this case, gain is not calculated, and no attempt is made to convert the results into the time domain.

Yet another work that seems to cover similar material is that by Neitz et al. [24]. They use a plane-wave scattering matrix theory to derive a version of the Friis transmission equation that is valid in both the near and far fields. However, they offer no equation analogous to (5) of this article, nor do they show how to use signal-flow graphs to solve more complicated problems. Finally, they do not show how to handle the time domain.

CONCLUSIONS

The antenna equation solves a remarkable array of antenna-theory puzzles. It defines the time domain analog of gain, so one can compare the time domain performance of various antennas. It shows how to combine gain with a meaningful phase. It generalizes expressions so they work naturally with waveguide feeds. It allows one to reformulate the Friis transmission equation into a power wave expression that includes both magnitude and phase. It also works naturally with signal-flow graphs to easily solve complicated problems.

The antenna equation also solves other fundamental problems. It shows how to combine RCS with a meaningful phase, and it defines the time domain analog of the RCS. It explains how to represent coupling into and radiation from shielded enclosures. It shows how to describe the bandwidth of an antenna, transient antenna patterns, and mutual coupling in phased arrays. Finally, it explains why realized gain should be preferred over antenna gain in publications.

One consequence of the antenna equation is that the most important parameter of any antenna is the antenna transfer function, $\tilde{h}$, and its inverse transform, the antenna impulse response, $h(t)$. Remarkably, these parameters appear in almost no antenna textbooks. These parameters should be added to the next revision of the antenna definitions standard [1]. Their widespread use should be encouraged.

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