MINIMIZING DISPERSION IN A TEM WAVEGUIDE BEND BY A LAYERED APPROXIMATION OF A GRADED DIELECTRIC LENS

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INTRODUCTION

Waveguide bends pose a problem for high-voltage UWB systems or for any transmission-line system with low-loss/fast-risetime requirements. The difficulty arises because only a straight section of conventional waveguide can support the pure TEM mode necessary to preserve the risetime of a transmitted pulse. When there is a bend in the waveguide, especially when the cross-section of the waveguide is large, as it typically must be for high-voltage or low-loss systems, the risetime of the transmitted signal is lengthened. This can severely limit the system bandwidth.

In several papers, [1, 2, 3, 4, 5, and 6], Dr. Carl E. Baum has described an approach that addresses this problem. We described our initial implementation of some of those ideas in [7]. This paper is an abbreviated description of the work originally reported there, and the reader is referred to that document for additional detail.

Conventional waveguide is filled with a *homogeneous* dielectric material, sometimes air, or is evacuated. Here we consider compensation of a waveguide bend by a purely dielectric lens, characterized by an *inhomogeneous*, frequency-independent, isotropic permittivity. If the relative index of refraction of the lens varies inversely as the radius of curvature, then the optical path length through the bend will be independent of the radius, and the bend will support pure TEM waves.

Although waveguide structures employing lens materials having the required inhomogeneous permittivity profile are easily conceptualized, constructing them is more problematic. In this paper, we describe an approximation employing coarsely graded layers of dielectric materials of various uniform permittivities. We used this approach to compensate a 90° H–plane bend in a strip transmission line.

Here, we first summarize the relevant theory. Then, we describe our hardware implementation of the graded dielectric strip line bend. We analyzed the structure numerically, and we determined its performance experimentally. The measured pulse risetime was 75 ps for a straight air section of strip line. For a 90° air bend, it was 255 ps.

With our layered compensating lens installed, the risetime of the transmitted pulse was reduced to 185 ps. By thus reducing the dispersion of a transmitted pulse, the strip line bend provided a proof-of-principle demonstration of compensation of a waveguide bend by a graded dielectric lens.

CONCEPT

Consider the waveguide bend shown in Figure 1A, which consists of a strip transmission line above a ground plane, all in air. The straight sections of the waveguide are known to support the non-dispersive propagation of a pure TEM wave, provided that the conductors



Figure 1. A conventional waveguide bend is dispersive because the path length around the bend is proportional to the radius of curvature (A). A bend may be compensated with a graded dielectric material (B). If the index of refraction within the bend varies inversely as the radius of curvature, optical path lengths through the bend are equalized.

are perfectly conducting. However, the curved section cannot support a pure TEM wave, since the path length through the bend is proportional to the radius of curvature. Rays entering the bend at its outer edge must travel farther than those entering at the inside edge. As a result, they lag behind those at the inside edge. What is needed is a way to equalize the transit times for all rays propagating through the bend. Equivalently, we require that the optical (or electrical) path length through the bend be independent of the radius of curvature. Since the wave speed is inversely proportional to the index of refraction, a conceptually simple solution is to embed the bend in a dielectric medium, the index of which varies inversely with the radius of curvature, as

$$n(\Psi) = \Psi_{\rm max} / \Psi \tag{1}$$

where Ψ is the cylindrical radius of curvature. It is assumed that at some outer radius, Ψ_{max} , the fields no longer contribute significantly, and the index reaches a minimum of

unity. Since the square of the index of refraction is the relative permittivity, we can also express (1) as

$$\varepsilon_r(\Psi) = \varepsilon(\Psi)/\varepsilon_0 = \left(\frac{\Psi_{\max}}{\Psi}\right)^2$$
 (2)

where ε_0 is the permittivity of free space. This approach to solving the waveguide bend problem is indicated conceptually in Figure 1B.

IMPLEMENTATION

We considered several alternative hardware implementations of non-dispersive waveguide bends. Ultimately, we selected a curved strip line embedded within graded dielectric layers machined from sheets of low-loss uniform-permittivity plastics. Use of this layered approximation of a graded dielectric material represented a compromise between modeling fidelity and ease of construction. Other possible implementations are discussed in [7].

Our test fixture layout is depicted in Figure 2. Details of the dielectric material layout in the bend region are shown in Figure 3. With the straight section in place, measurements were made between Port 1 and Port 2. With the curved section, measurements were made between Port 1 and Port 3. The curved section was used both with and without the layered graded dielectric lens.



Figure 2. Strip line test fixture layout. The wave launchers and straight strip section were always airfilled. The curved strip section was used either with or without the graded dielectric lens in place.

To find the characteristic impedance of the graded dielectric structure, one first solves a two-dimensional equation in cylindrical coordinates, in a plane of constant azimuth, ϕ . In



Figure 3. Five-region graded dielectric bend assembly. Straight sections at the entrance and exit of the bend are not shown. The conducting strip is 1.27 cm (0.5 in.) above the ground plane. Its width in both straight and curved regions is a constant 6.35 cm (2.5 in.). Within the strip boundaries, the relative permittivities approximate an inverse square law relationship with the radius of curvature. The relative permittivity is assumed to be 1.0 at a radius of 27.94 cm (11.0 in.). All drawing dimensions are inches.

terms of the electric potential, $V(\Psi, z)$, the equation to be solved is [7]

$$\Psi \nabla \bullet \left(\frac{1}{\Psi} \nabla V\right) = 0 \tag{3}$$

where the boundary conditions are $V = V_0$ on the conducting strip and V = 0 on the ground plane. The permittivity, ε , varies inversely as the square of the radius of curvature in the bend region, as specified previously in (2). As an approximation, we used the finite element method to solve a simpler problem. We assumed that the change in azimuth through the bend could be ignored—a straight strip approximation. This allowed Ψ and z to be treated as Cartesian coordinates, and reduced the problem to solving Laplace's equation,

$$\nabla \bullet \left(\varepsilon_i \ \nabla V \right) = 0 \tag{4}$$

over a rectangular (Ψ, z) domain consisting of sub-domains having piecewise-uniformpermittivities. As described in [7], where the straight strip approximation is justified, this method produced a characteristic impedance of 28 Ω , in agreement with a TDR measurement of the bend (Figure 4).



Figure 4. TDR impedance measurement of the strip line test fixture with a 90° bend embedded within a five-layer approximation of a graded dielectric lens. Within the bend region, the impedance is approximately 28 Ω . Note that the discontinuities between the feed sections and the bend could be reduced by use of a dielectric with the same permittivity as the average permittivity within the bend region.

In Figure 5, transmission of a voltage step pulse through the straight, air-filled strip line section (between Port 1 and Port 2), is compared with transmission of the same pulse through the curved section (between Port 1 and Port 3), both air-filled and compensated by the five-layer graded dielectric lens. Transmission through the straight strip line section (and the feed sections) degraded the fall time to 75 ps. This, then, is the best we can expect for a perfectly compensated curved section. The air-filled bend degraded the fall time to 255 ps. The graded dielectric layers improved the transmission through the bend, reducing the fall time to 185 ps.



Figure 5. Transmission of a voltage step by straight and bent strip lines. The fall time for the straight line is about 75 ps. With the air-filled, 90° bend, the fall time increases to about 255 ps. For the same bend with a five-layer radially graded dielectric lens, the fall time is reduced to about 185 ps. The data are normalized to correct for reflection losses at the dielectric-air interfaces.

The primary causes of the imperfect compensation of the bend appear to lie in the finite thickness of the dielectric layers, and in the permittivities of the materials filling the fringe field regions above and on each edge of the strip. The difference in transit time around the bend at the strip edges is 330 ps for the air bend; for the layered dielectric bend, the difference is only 110 ps. However, the transit time difference for the dielectric bend climbs to 350 ps when fringe regions 1.3 cm on each side of the strip may also contribute. If we calculate the transit times along each dielectric layers immediately adjacent to each interface range from a low of 74 ps to a high of 170 ps. The aggregate effect of these transit time differences is to stretch the transmitted pulse.

CONCLUSIONS

Although the five-layer graded dielectric bend substantially improved the transmission of a stepped voltage pulse through the curved section of our strip line test fixture, residual variations in bend transit time limited the improvement that could be achieved. Thinner dielectric layers, with well-characterized indices of refraction, as well as use of a closed structure, instead of an open strip line, will be required if the potential of this technique for minimizing dispersion in waveguide bends is to be fully realized.

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