Switching Notes

Note 30

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A Test Chamber for a Gas Switch Using a Hyperboloidal Lens

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Abstract

We consider here a test chamber for measuring the output of a gas switch with fast risetime. This test chamber is being used to experiment with a new switch called the ferratron, which provides a high-voltage output at low jitter and fast repetition rate (L. H. Bowen, *et al*, Switching Note 29, 1999).

The test chamber is designed to launch a fast-risetime wave into an electrically large coaxial structure. The transition from gas to oil is accomplished with a polyethylene lens with one surface being a hyperbola of revolution, or hyperboloid. The goal of the transition is to preserve the risetime of the wave as it transitions from gas to oil coax. Since the output coax is electrically large, optical methods are required to maintain a planar phase front in the coaxial output.

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I. Introduction

We consider here a test chamber that is suitable for measuring the output of high-voltage switches with fast risetimes in a closed configuration. We are considering voltages as high as 50-100 kV and risetimes as fast as 100 picoseconds. The chamber is designed in such a way that it should have little, if any, effect on the measured risetime of the gas switch. This test chamber is being used to study the next generation of the ferratron switch [1]. A typical switch with the hyperboloidal lens is shown in Figure 1.1.

The challenges encountered in this problem are ones we have seen before. High voltages push us toward electrically large designs, while fast risetimes demand smaller dimensions. To compromise, we design a test fixture that forces all ray paths from the point spark gap to any position on a planar cross section of the coax line to be a constant. Furthermore, we require that the coaxial geometry be immersed in oil or some low-loss dielectric with an equivalent dielectric constant. The spark gap, is in gas, so a transition is necessary between the two media.

The transition section works as follows. After launching the wave into the biconical region, the hyperboloidal lens is used to straighten the wavefront so a planar phase front is launched onto the coaxial line. To obtain the best possible measurement of the spark characteristics, one can place a sensor in the outer conductor of the coax. Alternatively, one can taper the coax down at constant impedance to feed into a flexible coax line, which then is used to feed another device, such as an antenna.

The geometry that satisfies all of the above requirements is a lens with a hyperbola of revolution (hyperboloid). A further constraint on the design is that the biconical feed line in gas must have the same impedance as the coaxial output in oil. Typically, one would want both transmission line sections to be 50 ohms. This constraint forces a single choice of all angles, for a given ratio of dielectric constants between oil and gas.

We begin by deriving the equations of the hyperboloid. Next, we impose the impedance matching condition. Finally, we show an example of a test chamber that is currently in development.

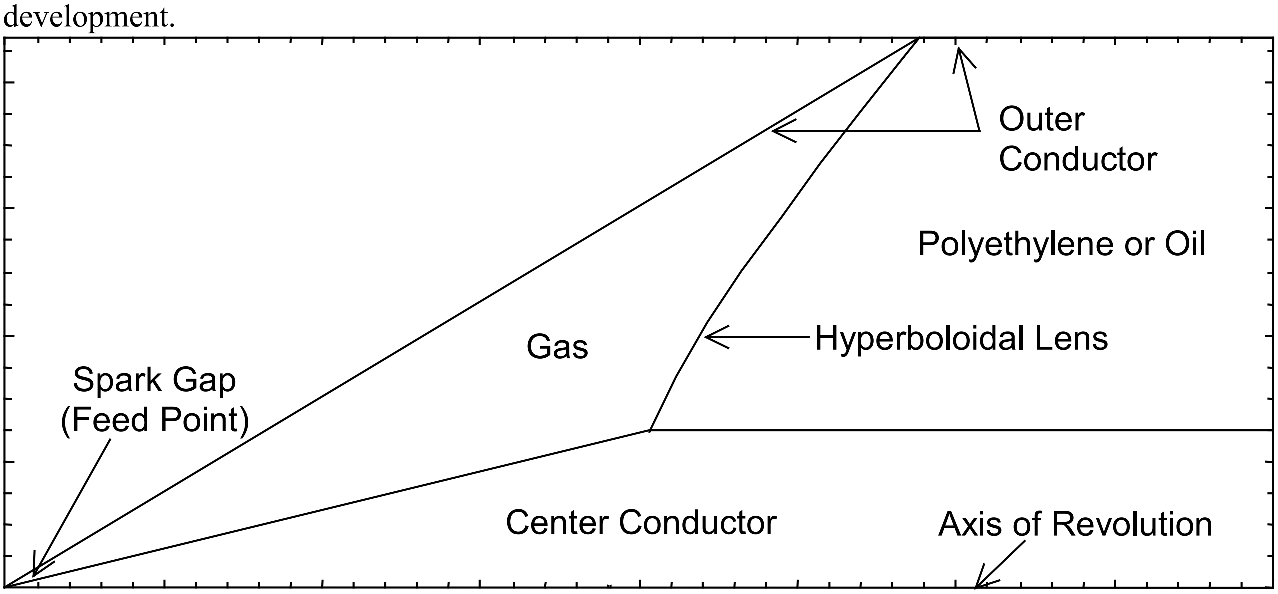

Figure 1.1. A transition from a biconical line in gas to a coaxial line in polyethylene or oil.

II. Hyperboloidal Lens Derivation

We seek here to transition smoothly from a point spark gap to an electrically large coaxial cable. A similar problem has already been solved in [2], so we can use techniques developed in that paper. In [2], the authors developed a transition from a biconical feed section with high dielectric constant into a coaxial geometry of a relatively lower dielectric constant. The resulting lens shape was prolate spheroidal (an ellipse of revolution). The only difference between the problem of [2] and the problem of this paper is that we require a transition from low to high dielectric constant. So the lens shape is hyperboloidal, or a hyperbola of revolution.

To derive the equation of the hyperbola, consider the configuration of Figure 2.1. All rays emanating from the apex at $(z,\Psi) = (0{,}0)$ and arriving at the dotted line must have an equal electrical length. Furthermore, an arbitrary ray intersecting the hyperbola at (z,Ψ) must have the same electrical length as the ray at goes straight through, on the axis of rotation. By equating the electric lengths of the two ray paths we find

$$d + \sqrt{\varepsilon_r}\,(\ell - d) = \sqrt{z^2 + \Psi^2} + \sqrt{\varepsilon_r}\,(\ell - z) \tag{2.1}$$

This simplifies a bit to

$$\begin{aligned} \sqrt{\varepsilon_r}\, z - q &= \sqrt{z^2 + \Psi^2} \\ q &= d\left(\sqrt{\varepsilon_r} - 1\right), \qquad q > 0 \end{aligned} \tag{2.2}$$

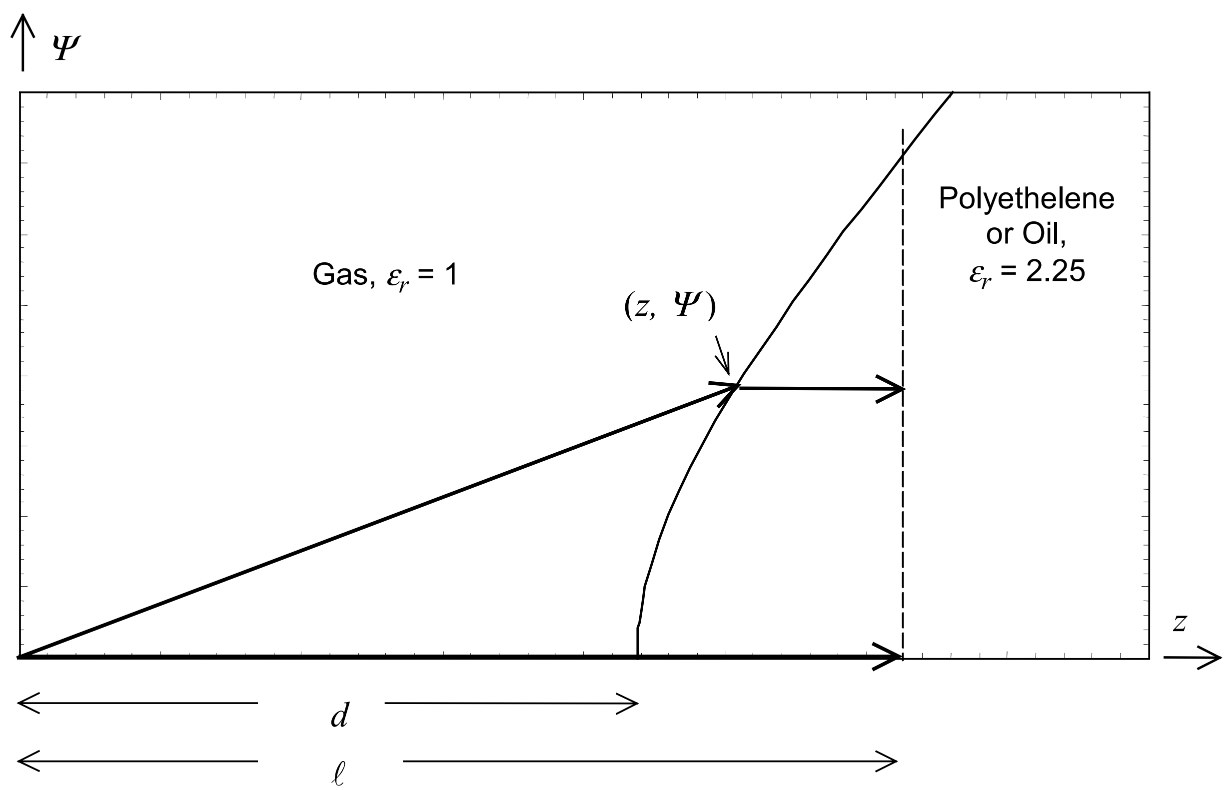

Figure 2.2. Rays required for deriving the equation of the hyperobola.

After squaring both sides and rearranging, we have the quadratic form

$$(\varepsilon_r - 1)\, z^2 \ - 2\sqrt{\varepsilon_r}\, q\, z \ + \ (q^2 - \Psi^2) \ = \ 0 \tag{2.3}$$

This is solved as

$$z \ = \ \frac{\sqrt{\varepsilon_r}\, q \ \pm \ \sqrt{\varepsilon_r\, q^2 \ - \ (\varepsilon_r - 1)(q^2 - \Psi^2)}}{\varepsilon_r - 1} \tag{2.4}$$

This fully describes the hyperbola of revolution that is required for the lens. The inner and outer conductors of both the biconical tapered feed and the coax must follow the a path through the material.

It is perhaps of some interest to express the hyperbola in a more normal form. Starting from (2.3), we can complete the square and rearrange terms to find

$$\begin{gathered} \frac{(z-p)^2}{p^2 \ - \ \dfrac{q^2}{\varepsilon_r - 1}} \ - \ \frac{\Psi^2}{(\varepsilon_r - 1)\, p^2 - q^2} \ = \ 1 \\ p \ = \ \frac{\sqrt{\varepsilon_r}\, q}{\varepsilon_r - 1} \ = \ \frac{\sqrt{\varepsilon_r}\, d}{\sqrt{\varepsilon_r} + 1} \ , \qquad\qquad p > 0 \end{gathered} \tag{2.5}$$

This is recognized as a hyperbola with focus at $(z, \Psi) = (p, 0)$ and asymptote at $\Psi = (\varepsilon_r - 1)\, z$.

Having described the hyperbola, we now recognize that a given ratio of dielectric constants leads invariably to a given shape of the lens. However, we still have to match the impedance of the biconical taper to the coaxial line. In other words, we still have to choose a value for d. We implement the impedance matching in the next section.

III. Impedance Matching

A key parameter that we left undefined in the previous section is d, the distance from the feed point to the apex. This parameter must be chosen so that both the biconical taper and the coaxial line have the same characteristic impedance. We provide here the equations that must be solved to enforce the continuity of impedance in the two sections of transmission line.

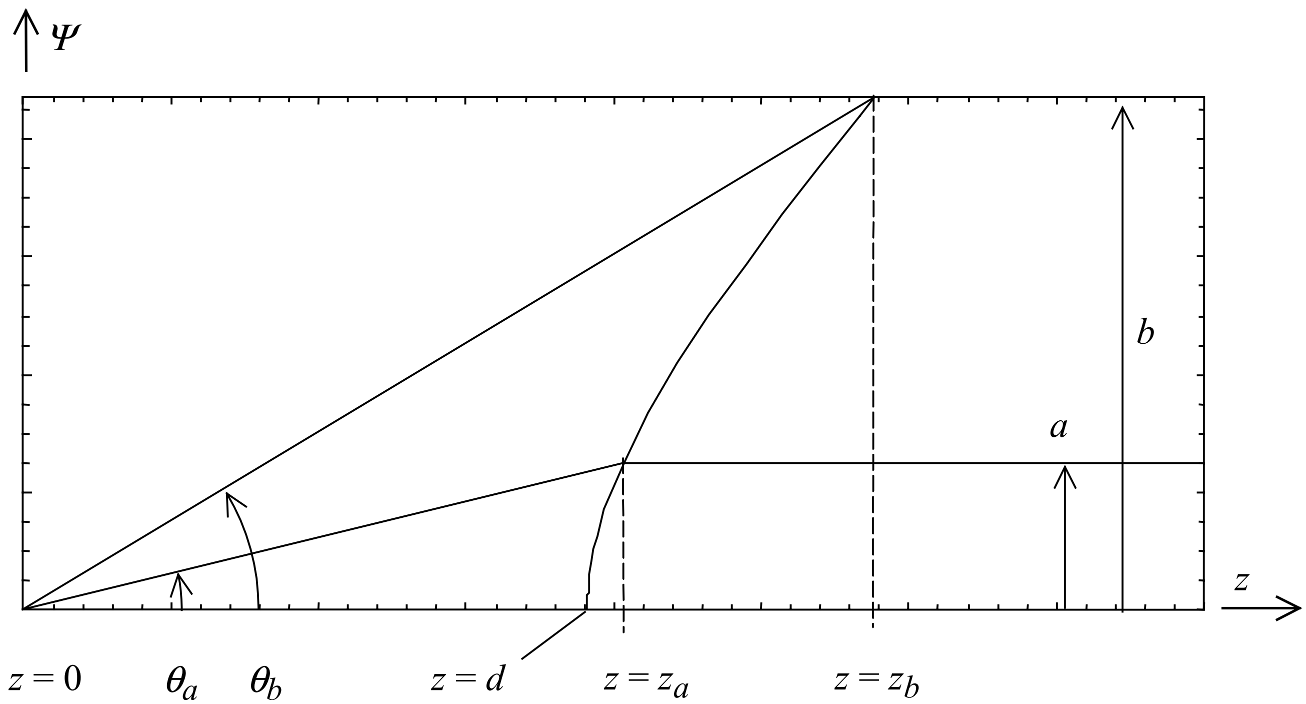

Figure 3.1 Geometry of the transition region.

The relevant geometry is shown in Figure 3.1. The impedance of the biconical feed is

$$Z_1 = \frac{\eta}{2\pi} \ln\left[\frac{\tan(\theta_b/2)}{\tan(\theta_a/2)}\right] \tag{3.1}$$

where η is the impedance of free space, 376.727 Ω. The impedance of the coax is

$$Z_2 = \frac{\eta}{2\pi\sqrt{\varepsilon_r}} \ln(b/a) \tag{3.2}$$

Since we want the two impedances to be equal, we have

$$\frac{\tan(\theta_b/2)}{\tan(\theta_a/2)} = \exp^{\left[\ln(b/a)/\sqrt{\varepsilon_r}\right]} \tag{3.3}$$

At this point, we normally know what outer radius we want for the coax, and the required impedance (usually 50 Ω). Thus, we know a and b. We also know the dielectric constant of the higher dielectric material, ε_r. So the only unknowns are the two angles in the above equation.

To find the angles, we have from simple geometry and from (2.4)

$$\tan(\theta_a) = \frac{a}{z_a}, \qquad z_a = \frac{\sqrt{\varepsilon_r}\, q + \sqrt{\varepsilon_r q^2 - (\varepsilon_r - 1)(q^2 - a^2)}}{\varepsilon_r - 1}$$
$$\tan(\theta_b) = \frac{b}{z_b}, \qquad z_b = \frac{\sqrt{\varepsilon_r}\, q + \sqrt{\varepsilon_r q^2 - (\varepsilon_r - 1)(q^2 - b^2)}}{\varepsilon_r - 1} \tag{3.4}$$

Equations (3.3) and (3.4) are three simultaneous equations in three unknowns, θ_a and θ_b, and q. These are solved numerically by a root-finding algorithm, and d is found from $d = q/(\sqrt{\varepsilon_r} - 1)$. This completes the solution of the problem.

A typical solution, when normalized to outer radius, is $\varepsilon_r = 2.25$, $b = 1$, $a/b = .286254$, $q/b = .54705$, $z_a = 1.16350$, $z_b = 1.65222$, $\theta_a = 13.82^\circ$, $\theta_b = 31.18^\circ$. We are currently building a test chamber with these dimensions, with center radius set to 6.35 mm (0.25 in). Thus, the dimension of the actual device are $b = 0.873351$ in, $a = 0.25$ in, $q = 0.47777$ in, $d = 0.95541$ in, $z_a = 1.01614$ in, $z_b = 1.44297$ in, $\theta_a = 13.82^\circ$, and $\theta_b = 31.18^\circ$.

Next, we run a parameter study of the solution. We choose the case where $\varepsilon_r = 2.25$ and $Z_2 = 50$ Ω to be the central case. To run the parameter study, we vary one of these two parameters, while keeping the other fixed. First, we consider the case with Z_2 fixed at 50 Ω, and allowing ε_r to vary. To specify a complete solution, it is sufficient to plot only q/b, θ_a and θ_b. These results are shown in Figure 3.2. Next, we vary Z_2, while holding ε_r fixed. The results are shown in Figure 3.3.

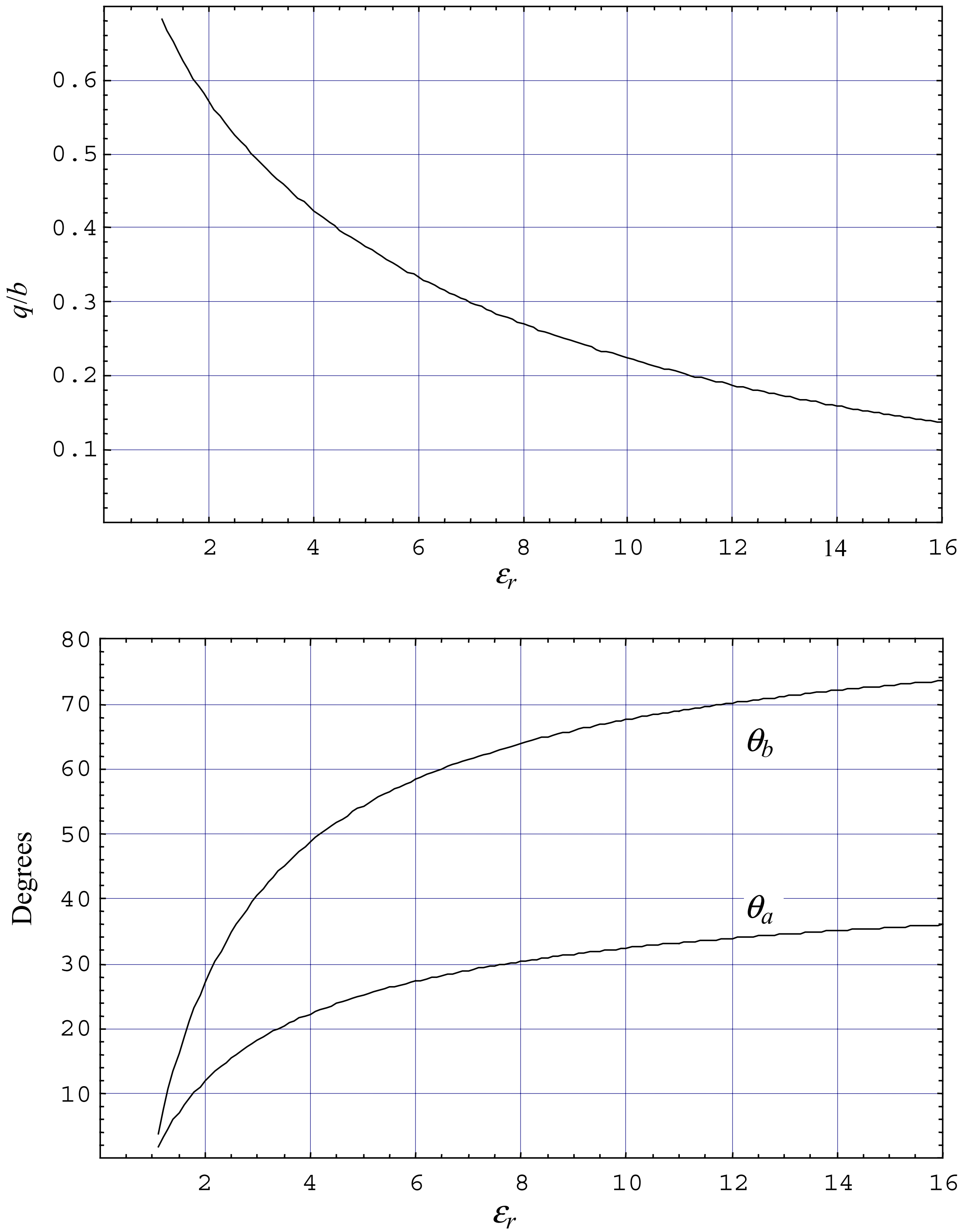

Figure 3.2. Parameter study of the variation of q/b(top), and θ_a and θ_b (bottom), while varying ε_r and keeping Z_2 fixed at 50 Ω.

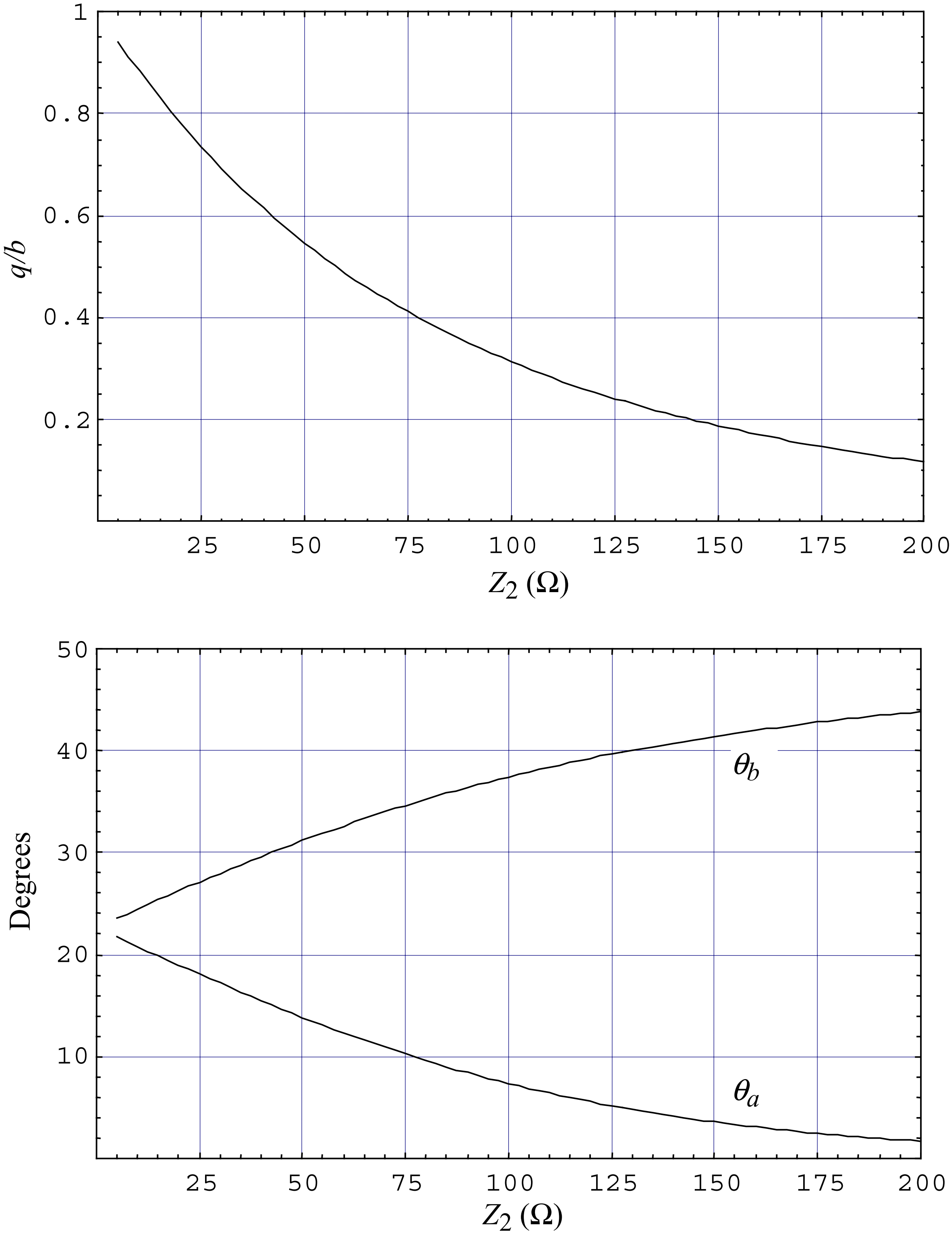

Figure 3.3. Parameter study of the variation of q/b(top), and θ_a and θ_b (bottom), while varying Z_2 and keeping ε_r fixed at 2.25.

IV. A Typical Application, Including Calibration

A typical application of how such a device might be used is shown in Figure 4.1. The high-voltage switch is located at the apex of the tapered section. A field probe is located at the location labeled as “sensor” in the diagram. A measurement of the field at the sensor is expected to provide an accurate replication of the fields in the conical tapered section.

In order to calibrate the test fixture and sensor combination, we also plan to build an alternative feed point, as shown in Figure 4.2. This alternative feed point will allow us to drive the feed point directly with a fast-risetime source (calibration pulse), and measure the output at the sensor. This is essentially a Time Domain Transmission (TDT) measurement through the test fixture. Assuming the calibration pulse has a voltage into a 50-ohm load of $V_{cal,src}(t)$, and the output at the sensor as $V_{cal,out}(t)$, then we have the data required to correct our measurements of the switch. If our raw measurement is $V_{raw}(t)$, then we can Fourier transform all three waveforms and take the ratio, to find a corrected waveform as

$$V_{cor}(\omega) = \frac{V_{cal,src}(\omega)}{V_{cal,out}(\omega)} V_{raw}(\omega) \tag{4.1}$$

We now need only transform $V_{cor}(\omega)$ back into the time domain to obtain the corrected time domain measured waveform.

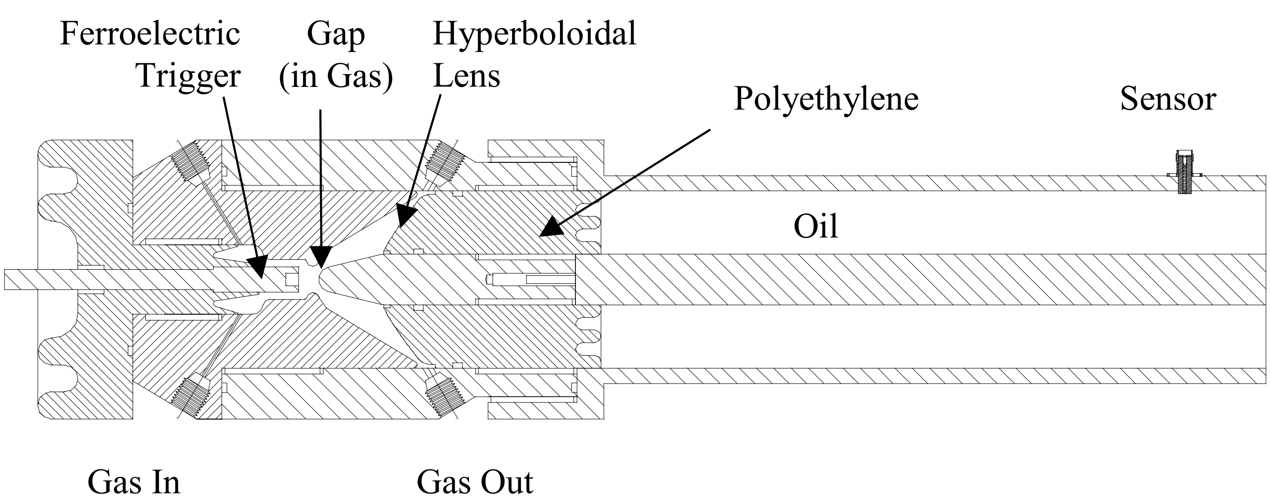

Figure 4.1. A typical implementation of the hyperboloidal lens, including a sensor.

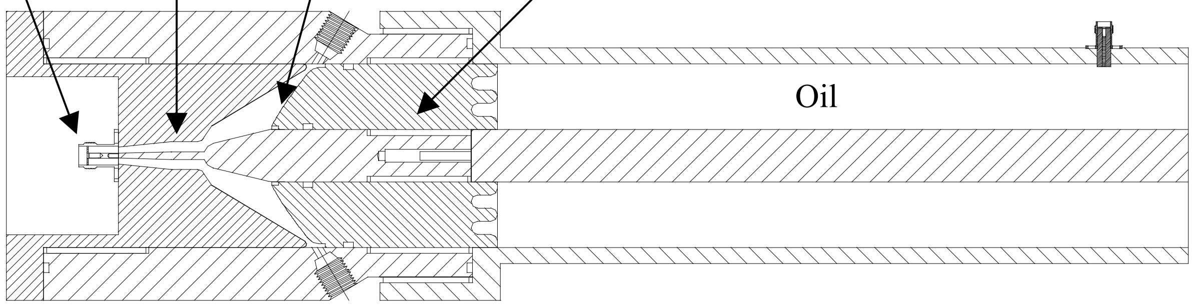

Figure 4.2. Configuration for calibrating the test chamber, using a coaxial SMA feed at the apex.

V. Conclusion

We have designed a test chamber that can be used to study the behavior of gas switches. Such a test chamber can be used to measure precisely the risetime of a gas switch in a closed configuration, while introducing only minimal degradation of risetime. This design is currently being used to test the newest variations of the ferratron, and could be used to study any gas switch.

References

1. L. H. Bowen, E. G. Farr, J. M. Elizondo, and J. Lehr, The Ferratron: A High-Voltage, High Rep-Rate, Low Jitter, UWB Switch with A Ferroelectric Trigger, Switching Note 29, March 1999; *IEEE Pulsed Power Conference Proceedings*, Monterrey, 1999.

2. C. E. Baum, J. J. Sadler, and A. P. Stone, A Prolate Spheroidal Uniform Isotropic Dielectric Lens Feeding a Circular Coax, Sensor and Simulation Note 335, December 1991; *Electromagnetics*, 1995, pp. 223-241.