Sensor and Simulation Notes

Note 555

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A Standard for Characterizing Antenna Performance in the Time Domain

(With Corrections)

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Abstract

We derive here a simple function describing antenna performance in the time domain. This function describes antenna performance in both transmission and reception, and in both the time and frequency domains. The resulting equations are as simple as possible. From this function one can simply derive such conventional frequency domain quantities as gain, realized gain, and antenna factor. It is hoped that this function will be adopted as an IEEE standard for time domain antenna performance.

I. Introduction

There are already standards for characterizing antenna performance in the frequency domain, but no such standard exists in the time domain. This becomes a challenge if, for example, one wishes to buy or sell an antenna with a specified performance in the time domain. In the frequency domain, one normally uses antenna gain [1], but this offers little information about the antenna's time domain performance.

A number of earlier papers have addressed this issue [2-5]. However, there remains no standard method of describing antenna performance in the time domain. We demonstrate here a method of simplifying the equations as much as possible, leading to a standard waveform describing antenna performance.

In order to characterize an antenna in the time domain, a function should have five characteristics.

1. The function should fully describe antenna performance with equations that are as simple as possible.

2. The function should describe antenna performance in both transmission and reception.

3. The function should describe antenna performance in both the frequency and time domains.

4. The function should be simply related to frequency domain standards, such as gain, realized gain, and antenna factor.

5. The function should be simply related to quantities that are measurable in the laboratory, typically with an oscilloscope.

In this spirit, we propose the function $h_N(t)$, which satisfies all five criteria. This function was first derived in [5] for antennas with TEM feeds, but we extend the concept here to all antennas.

We begin by introducing the function of interest, $h_N(t)$, in Section II, along with the relevant antenna equations. At first, it is convenient to manipulate antenna equations that have been specialized to radiation on boresight with dominant polarization. We prove the antenna equations using $h_N(t)$ in Section III, using an argument of reciprocity. We extend the analysis to alternative source and load impedances in Section IV. We generalize the results to all angles and polarizations in Section V. We show the relationship between $h_N(t)$ and other commonly used antenna terms in Section VI. Finally, we discuss candidate names for $h_N(t)$ in Section VII.

II. The Proposed Function: $h_N(t)$

We provide the conclusion in this section, and justify it in the remainder of the paper.

We have found that the antenna equations exhibit a striking simplicity and symmetry if they are expressed not in terms of electric fields and voltages, but in terms of the square-root of power or power density. Thus, instead of voltages, we use voltages divided by the square root of the load or source impedance; and instead of electric fields, we use electric fields divided by the square root of the intrinsic impedance of free space. In this format, we have the following equations for transmission and reception on boresight, with dominant polarization,

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{377\,\Omega}} &= \frac{1}{2\pi c r}\, h_N(t) \circ \frac{d V_{src}(t')/dt}{\sqrt{50\,\Omega}} \,, \quad t' = t - r/c \\ \frac{V_{rec}(t)}{\sqrt{50\,\Omega}} &= h_N(t) \circ \frac{E_{inc}(t)}{\sqrt{377\,\Omega}} \end{aligned} \tag{2.1}$$

where $V_{rec}(t)$ is the received voltage into a 50-Ω load or oscilloscope, and $V_{src}(t')$ is the source voltage in retarded time as measured into a 50-Ω load or oscilloscope. Furthermore, $E_{inc}(t)$ is the incident electric field, $E_{rad}(t)$ is the radiated electric field, r is the distance away from the antenna, c is the speed of light in free space, and "∘" is the convolution operator. Note also that $h_N(t)$ has units of meters per second in the time domain, and meters in the frequency domain.

The above expressions have been simplified in three ways. They refer only to dominant polarization, they are valid only on boresight, and attenuation from source to receiver has been ignored. All three effects can be easily restored, and we do so in Section V. However, these effects do not affect the derivation, and we find it easier to manipulate simpler versions of the equations.

All five criteria cited in the previous section are met with $h_N(t)$. The above equations are as simple as possible. The function $h_N(t)$ describes antenna performance in both transmission and reception, and in both the frequency and time domains. The function $h_N(t)$ is simply expressed in terms of voltages that are measured with an oscilloscope. And finally, as we will see later, $h_N(t)$ is simply related to gain, realized gain, and antenna factor.

III. Derivation of $h_N(t)$ Using Reciprocity

To derive the antenna equations in terms of $h_N(t)$, we invoke reciprocity. At first, we follow the development of Baum [2], and then we add modifications later. It is simplest to derive the equations in the frequency domain, and then convert to the time domain at the end.

We consider first the equations of an antenna in transmission. We consider three cases for driving the antenna: open circuit voltage, $\tilde{V}$, short circuit current, $\tilde{I}$, and a loaded source voltage, $\tilde{V}_s$, with source impedance $\tilde{Z}_s$, as shown on the left in Figure 3.1. The tildes indicate a dependence on frequency. The antenna's input impedance is $\tilde{Z}_{in}$, under the assumption that the transmitting and receiving antennas are positioned far away from each other.

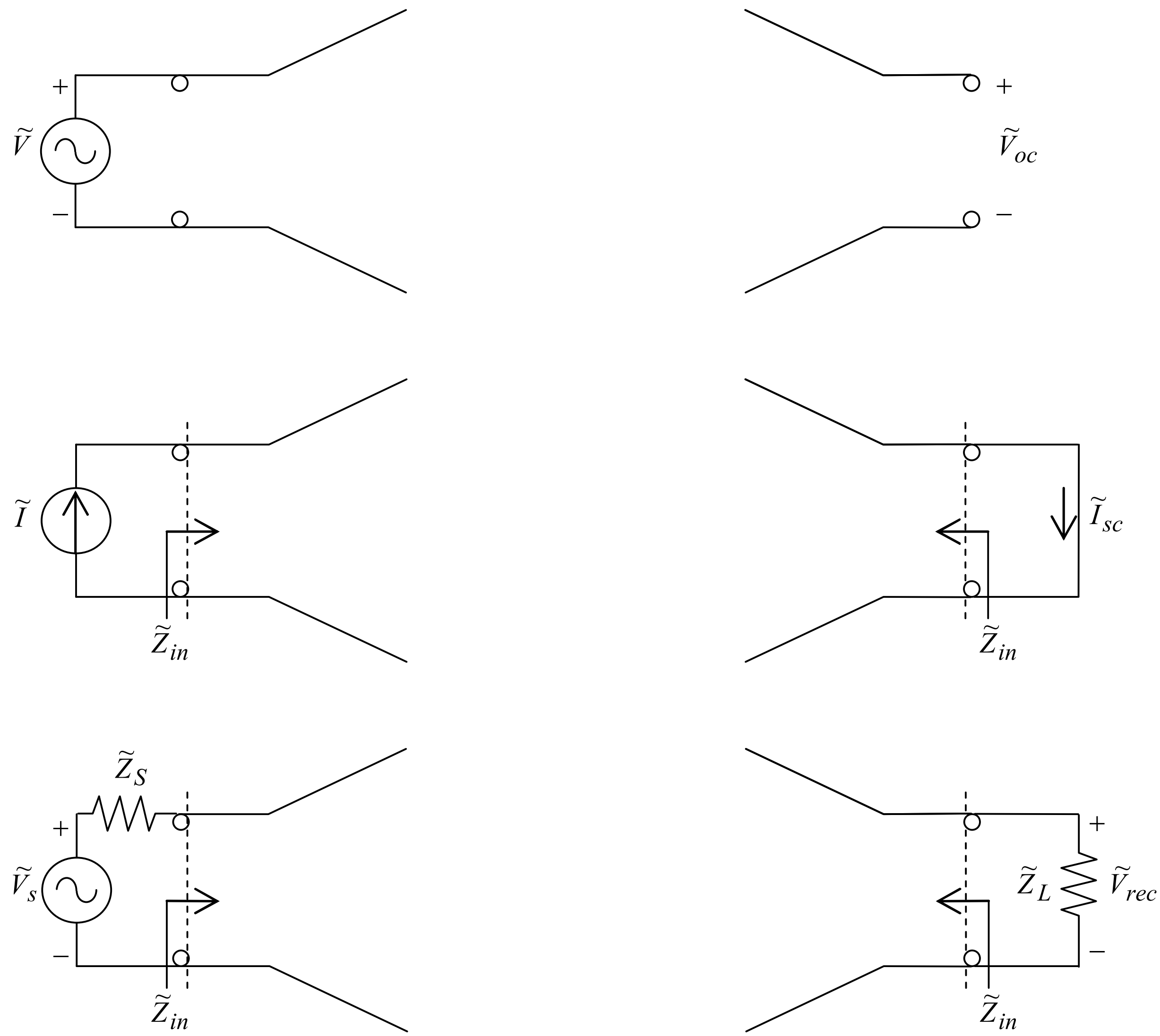

Figure 3.1. Three cases of antenna transmission (left) and reception (right) under conditions of open circuit (top), short circuit (middle), and load (bottom).

The equations in transmission may be expressed in one of three forms, depending on the source type.

$$
\begin{aligned}
\tilde{E}_{rad} &= \frac{e^{-jkr}}{r} \tilde{F}_V \; \tilde{V} \\
&= \frac{e^{-jkr}}{r} \tilde{F}_I \; \tilde{I} \quad , \qquad \tilde{F}_I = \tilde{Z}_{in} \; \tilde{F}_V \\
&= \frac{e^{-jkr}}{r} \tilde{F}_w \, \tilde{V}_s \quad , \qquad \tilde{F}_w = \frac{\tilde{Z}_{in}}{\tilde{Z}_{in} + \tilde{Z}_s} \tilde{F}_V
\end{aligned}
\tag{3.1}
$$

where $\tilde{E}_{rad}$ is the radiated far field, and $k = \omega / c = 2\pi f / c$ is the propagation constant. As mentioned earlier, for reasons of simplicity we removed the usual dependencies on angle, polarization, and attenuation, which we will restore later.

Next, we consider the antenna equations in reception. As before, we consider three cases: open circuit voltage, short circuit current, and voltage across a load; as shown on the right in Figure 3.1. In these cases we have

$$
\begin{aligned}
\tilde{V}_{oc} &= \tilde{h}_V \; \tilde{E}_{inc} \\
\tilde{I}_{sc} &= \tilde{h}_I \; \tilde{E}_{inc} \quad , \qquad \tilde{h}_I = \frac{1}{\tilde{Z}_{in}} \; \tilde{h}_V \quad . \\
\tilde{V}_{rec} &= \tilde{h}_w \; \tilde{E}_{inc} \quad , \qquad \tilde{h}_w = \frac{\tilde{Z}_L}{\tilde{Z}_{in} + \tilde{Z}_L} \tilde{h}_V
\end{aligned}
\tag{3.2}
$$

Note that we use the convention here that positive current flows into the load.

We now establish a relationship between the various forms of $\tilde{F}$ and $\tilde{h}$ using reciprocity. To do so, consider a two-port circuit consisting of the two-antenna system shown in Figure 3.2. Note that each antenna lies in the far field of the other. If we consider this to be just two ports of a linear time-invariant circuit, then it can be described in terms of Z- or Y-parameters, relating open circuit voltages on one port to short-circuit currents at the other. Furthermore, for a reciprocal system, $\tilde{Z}_{12} = \tilde{Z}_{21}$ and $\tilde{Y}_{12} = \tilde{Y}_{21}$, as shown, for example, in [6].

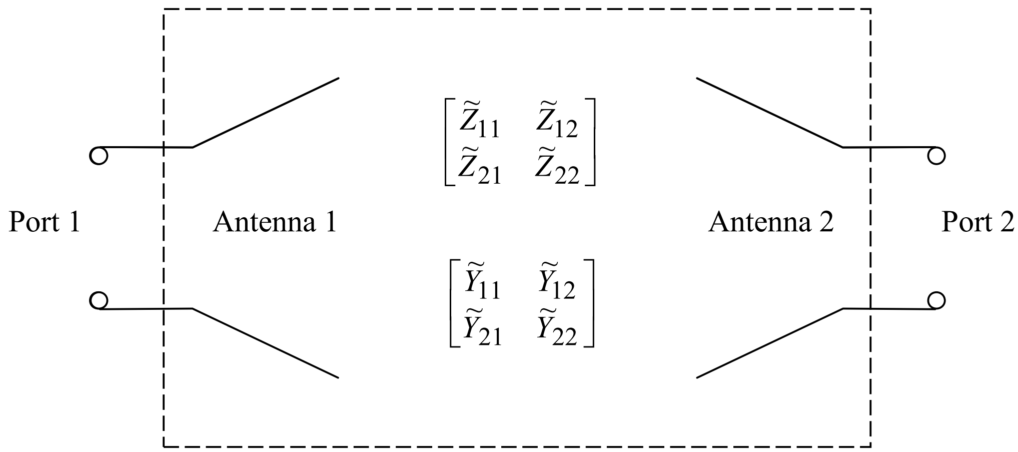

Figure 3.2. Two-port equivalent circuit of two-antenna system.

Thus, if we drive Port 1 with an open circuit voltage and measure the short-circuit current at Port 2, we get the same result if we switch ports. The two short-circuit currents are expressed as

$$\begin{aligned} \tilde{I}_{sc}{}^{(2)} &= \frac{e^{-jkr}}{r} \tilde{h}_I{}^{(2)} \tilde{F}_V{}^{(1)} \tilde{V}^{(1)} \\ \tilde{I}_{sc}{}^{(1)} &= \frac{e^{-jkr}}{r} \tilde{h}_I{}^{(1)} \tilde{F}_V{}^{(2)} \tilde{V}^{(2)} \end{aligned} \tag{3.3}$$

where the superscripts (1) and (2) specify the antenna. Since $\tilde{I}_{sc}{}^{(2)} = \tilde{I}_{sc}{}^{(1)}$ and $\tilde{V}^{(1)} = \tilde{V}^{(2)}$,

$$\frac{\tilde{F}_V{}^{(1)}}{\tilde{h}_I{}^{(1)}} = \frac{\tilde{F}_V{}^{(2)}}{\tilde{h}_I{}^{(2)}} \tag{3.4}$$

and since $\tilde{F}_V = \tilde{F}_I / \tilde{Z}_{in}$ and $\tilde{h}_I = \tilde{h}_V / \tilde{Z}_{in}$, we have

$$\frac{\tilde{F}_I{}^{(1)}}{\tilde{h}_V{}^{(1)}} = \frac{\tilde{F}_I{}^{(2)}}{\tilde{h}_V{}^{(2)}} . \tag{3.5}$$

These are the basic reciprocity relationships.

Let us now assume that Antenna #2 is a small canonical test antenna, for which it is easy to calculate $\tilde{F}_I$ and $\tilde{h}_V$ (or $\tilde{F}_V$ and $\tilde{h}_I$). The above equations say that if we know the ratio of

$\widetilde{F}_I$ to $\widetilde{h}_V$ (or $\widetilde{F}_V$ to $\widetilde{h}_I$) for our canonical test Antenna #2, we also know it for any arbitrary antenna under test, Antenna #1. The canonical test antenna can be, for example, an electrically small electric or magnetic dipole. For an electrically small electric dipole [2]

$$\widetilde{F}_I = \frac{j\omega\mu_o}{4\pi} h_e \quad , \qquad \widetilde{h}_V = h_e \tag{3.6}$$

where h_e is the effective height of the electrically small electric dipole. Similarly, for a electrically small magnetic dipole [2]

$$\widetilde{F}_I = \frac{(j\omega)^2 \mu_o}{4\pi c} A_h \quad , \qquad \widetilde{h}_V = \frac{j\omega A_h}{c} \tag{3.7}$$

where A_h is the effective area of the magnetic dipole. Note that to arrive at the above relationships from the forms shown in [2], we needed the relationships $\gamma = s/c$, $s = j\omega$, and $Z_o = c\,\mu_o$, where Z_o is the impedance of free space. Taking the ratios in the above two equations, we find for both small electric and magnetic dipoles

$$\frac{\widetilde{F}_I}{\widetilde{h}_V} = \frac{j\omega\mu_o}{4\pi} = \frac{\widetilde{F}_V}{\widetilde{h}_I}. \tag{3.8}$$

Because of the relationships in (3.4) and (3.5), this must be true for all antennas.

We can now find the ratio of the wave parameters, $\widetilde{F}_w$ and $\widetilde{h}_w$, by using the relationships in (3.1) and (3.2). Noting that

$$\widetilde{F}_w = \frac{\widetilde{Z}_{in}}{\widetilde{Z}_{in} + \widetilde{Z}_s} \widetilde{F}_V \quad \text{and} \quad \widetilde{h}_w = \frac{\widetilde{Z}_{in}\widetilde{Z}_L}{\widetilde{Z}_{in} + \widetilde{Z}_L} \widetilde{h}_I \tag{3.9}$$

we have

$$\frac{\widetilde{F}_w}{\widetilde{h}_w} = \frac{j\omega\mu_o}{4\pi} \frac{1}{Z_L} \frac{\widetilde{Z}_{in} + \widetilde{Z}_L}{\widetilde{Z}_{in} + \widetilde{Z}_s} \tag{3.10}$$

In the usual case of interest, the source and load impedances are equal, so

$$\widetilde{Z}_L = \widetilde{Z}_s = f_g\, Z_o. \tag{3.11}$$

Here, f_g is introduced as the ratio of the source or load impedance to the impedance of free space. The source and load impedances are normally 50 Ω, but there is no need to enforce that condition yet, as long as they are equal. Combining the above two equations,

$$\frac{\tilde{F}_w}{\tilde{h}_w} = \frac{j\,\omega}{4\pi\,c\,f_g}\ . \tag{3.12}$$

If we combine this with the radiation equation for $\tilde{F}_w$ in (3.1), we have

$$\tilde{E}_{rad} = \frac{j\,\omega}{4\pi\,c\,f_g}\,\frac{e^{-jkr}}{r}\,\tilde{h}_w\,\tilde{V}_s\,. \tag{3.13}$$

Let us now consider the source, $\tilde{V}_s$, when driven into a matched load, as shown in Figure 3.3. This is the configuration one uses to characterize the source with an oscilloscope.

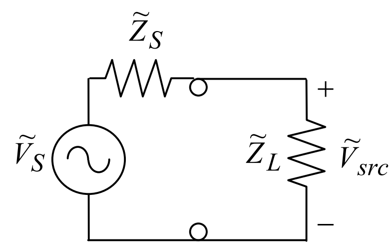

Figure 3.3.The source measurement, showing the relationship between $\tilde{V}_S$ and $\tilde{V}_{src}$.

For the normal situation of $\tilde{Z}_S = \tilde{Z}_L = f_g Z_o$,

$$\tilde{V}_S = 2\ \tilde{V}_{src}\,. \tag{3.14}$$

It is important to express our antenna equations in terms of $\tilde{V}_{src}$ instead of $\tilde{V}_s$, because it is $\tilde{V}_{src}$ that is actually measured with an oscilloscope, which normally has a 50-Ω input impedance. The transmit and receive equations in wave parameters then become

$$
\begin{aligned}
\tilde{E}_{rad} &= \frac{j\,\omega}{2\,\pi\,c\,f_g}\frac{e^{-jkr}}{r}\,\tilde{h}_w\,\tilde{V}_{src} \\
\tilde{V}_{rec} &= \tilde{h}_w\,\tilde{E}_{inc}
\end{aligned}
. \tag{3.15}
$$

We are now ready to introduce $\tilde{h}_N$. Let

$$
\tilde{h}_N = \frac{\tilde{h}_w}{\sqrt{f_g}} . \tag{3.16}
$$

Then

$$
\begin{aligned}
\tilde{E}_{rad} &= \frac{j\,\omega\,\mu_o}{2\,\pi\,c}\frac{e^{-jkr}}{r}\,\tilde{h}_N\,\frac{\tilde{V}_{src}}{\sqrt{f_g}} \\
\frac{\tilde{V}_{rec}}{\sqrt{f_g}} &= \tilde{h}_N\,\tilde{E}_{inc}
\end{aligned}
\tag{3.17}
$$

or, if we divide both sides of both equations by $\sqrt{Z_o}$,

$$
\begin{aligned}
\frac{\tilde{E}_{rad}}{\sqrt{Z_o}} &= \frac{j\,\omega}{2\,\pi\,c}\frac{e^{-jkr}}{r}\,\tilde{h}_N\,\frac{\tilde{V}_{src}}{\sqrt{f_g\,Z_o}} \\
\frac{\tilde{V}_{rec}}{\sqrt{f_g\,Z_o}} &= \tilde{h}_N\,\frac{\tilde{E}_{inc}}{\sqrt{Z_o}}
\end{aligned}
. \tag{3.18}
$$

Note that in the above equations both the electric field and the voltages are normalized to the square root of their local impedances. After doing so, $\tilde{h}_N$ describes both transmission and reception. For the normal case with a 50-Ω source and load impedances,

$$
\begin{aligned}
\frac{\tilde{E}_{rad}}{\sqrt{377\,\Omega}} &= \frac{j\,\omega}{2\,\pi\,c}\frac{e^{-jkr}}{r}\,\tilde{h}_N\,\frac{\tilde{V}_{src}}{\sqrt{50\,\Omega}} \\
\frac{\tilde{V}_{rec}}{\sqrt{50\,\Omega}} &= \tilde{h}_N\,\frac{\tilde{E}_{inc}}{\sqrt{377\,\Omega}}
\end{aligned}
\tag{3.19}
$$

or in the time domain, we get the result stated earlier in (2.1)

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{377\ \Omega}} &= \frac{1}{2\pi c r}\ h_N(t) \circ \frac{d V_{src}(t')/dt}{\sqrt{50\ \Omega}} \qquad t' = t - r/c \\ \frac{V_{rec}(t)}{\sqrt{50\ \Omega}} &= h_N(t) \circ \frac{E_{inc}(t)}{\sqrt{377\ \Omega}} \end{aligned} \tag{3.20}$$

where "$\circ$" is the convolution operator. This is the form of the antenna equation that has all the desirable characteristics listed in Section I.

On an antenna range we have two antennas. Combining the two equations for a transmitting and receiving antennas, we have

$$\frac{V_{rec}(t)}{\sqrt{50\ \Omega}} = \frac{1}{2\pi c r}\ h_{N,RX}(t) \circ h_{N,TX}(t) \circ \frac{d V_{src}(t')/dt}{\sqrt{50\ \Omega}} \qquad t' = t - r/c \tag{3.21}$$

where $h_{N,TX}(t)$ and $h_{N,RX}(t)$ are the $h_N(t)$ for the transmitting and receiving antennas, respectively.

The above equation suggests a simple method of characterizing an unknown antenna on an antenna range. If one uses two identical transmitting and receiving antennas (ones that are easily duplicated), and one measures the source and receive voltages, the only remaining unknown is the $h_N(t)$ of the identical transmit/receive antenna. After solving for the $h_N(t)$ of the standard antenna, one can then substitute an unknown antenna under test for one of the standard antennas, and find the $h_N(t)$ of the antenna under test. Additional detail is provided in [7, Appendix A].

Note that the normal measurement process assumes that both transmit and receive antennas are positioned in free space. Under this assumption, reflections from the ground, chamber walls, and support structures are excluded from $h_N(t)$ by using absorber material and/or with time gating. Under some circumstances, one might wish to include these extra features in the $h_N(t)$ of the antenna, and it might be reasonable to do so, as long as one is clear about how it was measured.

IV. The Effect of Alternative Source and Load Impedances on $h_N(t)$

As defined above, $h_N(t)$ is limited to the case where source and load are 50 Ω. This might not be immediately apparent from the form of equation (2.1), but it can be proven simply by calculating $h_N(t)$ for the simplest example antenna -- that of an electrically small electric dipole.

To find $h_N(t)$ for an electrically small electric dipole, we begin with equation (3.6),

$$\tilde{h}_V(\omega) = h_e \quad , \tag{4.1}$$

where all quantities are as defined earlier. When the antenna is loaded, we find

$$\tilde{h}_w(\omega) = \frac{\tilde{Z}_L}{\tilde{Z}_L + \tilde{Z}_{in}} \tilde{h}_V(\omega) = \frac{\tilde{Z}_L}{\tilde{Z}_L + \tilde{Z}_{in}} h_e \tag{4.2}$$

Using the definition for $\tilde{h}_N(\omega)$, we find

$$\tilde{h}_N(\omega) = \frac{\tilde{h}_w(\omega)}{\sqrt{f_g}} = \frac{\tilde{Z}_L}{\tilde{Z}_L + \tilde{Z}_{in}} \frac{h_e}{\sqrt{f_g}} \tag{4.3}$$

Thus, we see that for an electrically small electric dipole, $\tilde{h}_N(\omega)$ cannot be isolated from the load impedance. It will normally incorporate an assumption that the source and load impedance are 50 Ω. Since this is true of a simple antenna, it must be true of all antennas.

Using the above equations, it is straightforward to define a version of $h_N(t)$ for another impedance; say, a 75 Ω system. Let us call the new function $h_{N,75}(t)$. We now calculate the ratio of $h_{N,75}(t)$ to $h_{N,50}(t)$. We continue to use the example of the electrically small electric dipole, in order to determine the ratio. If we use (4.3) to calculate $h_N(t)$ for two difference load impedances, we have

$$\frac{\tilde{h}_{N,75}(\omega)}{\tilde{h}_{N,50}(\omega)} = \sqrt{\frac{75\ \Omega}{50\ \Omega}} \frac{50\ \Omega + \tilde{Z}_{in}}{75\ \Omega + \tilde{Z}_{in}} \tag{4.4}$$

Although this was derived for the electrically short electric dipole, it is true in general for all antennas. So we see that it is straightforward to generalize $h_N(t)$ to other impedance systems, if necessary. However, 50 Ω will likely be the most common one, and it will be understood to be the default.

V. Extension to More General Cases

For completeness, we now generalize the antenna equations by adding back in a dependence on angle, polarization, and attenuation. Thus, we have

$$
\begin{aligned}
\frac{\vec{E}_{rad}(\theta,\phi,t)}{\sqrt{377\,\Omega}} &= \frac{e^{-\alpha r}}{2\pi r c}\,\vec{h}_N(\theta,\phi,t) \circ \frac{dV_{src}(t')/dt}{\sqrt{50\,\Omega}} \quad , \quad t' = t - r/c \\
\frac{V_{rec}(t)}{\sqrt{50\Omega}} &= \vec{h}_N(\theta,\phi,t) \overset{\circ}{\bullet} \frac{\vec{E}_{inc}(\theta,\phi,t)}{\sqrt{377\,\Omega}}
\end{aligned}
\tag{5.1}
$$

where the "$\overset{\circ}{\bullet}$" operator is a dot-product convolution, and α is an attenuation constant, expressed in Np/m. Note that we have assumed that the attenuation constant is dispersionless, and does not vary with frequency. If there is a propagation loss with dispersion, then an additional convolution is required for the attenuation factor. The combined equation is then

$$
\begin{aligned}
V_{rec}(\theta,\phi,\theta',\phi',t) &= \frac{e^{-\alpha r}}{2\pi r c}\vec{h}_{N,RX}(\theta',\phi',t) \overset{\circ}{\bullet} \vec{h}_{N,TX}(\theta,\phi,t) \circ \frac{dV_{src}(t')}{dt} \\
t' &= t - r/c
\end{aligned}
\tag{5.2}
$$

where the primed angles refer to the orientation of the receiving antenna, and the unprimed angles refer to the orientation of the transmitting antenna.

VI. Gain, Realized Gain and Antenna Factor

Next, we demonstrate the relationship of $h_N(t)$ to realized gain, gain, and antenna factor. We derive realized gain from both the transmission and reception equations, and obtain the same result. Note that earlier derivations appeared in [7 (Appendix B), 8 (Appendix A)], however, the derivations here are simpler and more complete. Note that in those earlier works the term "effective gain" was used instead of the more appropriate (and IEEE standard) "realized gain."

A. Realized Gain Derived from the Receive Equation

We begin by deriving realized gain from the receive equation. From (2.1), the received voltage into a resistive load is

$$\frac{\tilde{V}_{rec}(\omega)}{\sqrt{f_g\, Z_o}} = \tilde{h}_N(\omega)\, \frac{\tilde{E}_{inc}(\omega)}{\sqrt{Z_o}} \tag{6.1}$$

If we multiply both sides of (6.1) by its complex conjugate, we get the received power as

$$\left|\tilde{P}_{rec}(\omega)\right| = \left|\tilde{h}_N(\omega)\right|^2 \left|\tilde{S}_{inc}(\omega)\right| \tag{6.2}$$

where $\tilde{S}_{inc}(\omega)$ is the incident power density. Alternatively, the received power is

$$\left|\tilde{P}_{rec}(\omega)\right| = \tilde{G}_r(\omega)\, \frac{\lambda^2}{4\pi} \left|\tilde{S}_{inc}(\omega)\right| \tag{6.3}$$

where $\tilde{G}_r(\omega)$ is the realized gain. If we now divide (6.3) by (6.2) we get

$$\tilde{G}_r(\omega) = \frac{4\pi}{\lambda^2}\, |\, \tilde{h}_N(\omega)\, |^2 = \frac{4\pi f^2}{c^2}\, |\, \tilde{h}_N(\omega)\, |^2 . \tag{6.4}$$

Note that $h_N(t)$ varies as the impedance system varies, as described in section IV. The impedance system will be assumed to be 50 Ω unless specified otherwise.

B. Realized Gain Derived from Transmission Equation

Alternatively, we may derive realized gain from the transmitted field. In transmission, realized gain is

$$\tilde{G}_r(\omega) = \frac{4\pi r^2}{\left|P_{avail}(\omega)\right|}\, \frac{\left|E_{rad}(\omega)\right|^2}{Z_o} \tag{6.5}$$

where P_{avail} is the power available to the antenna,

$$\tilde{P}_{avail}(\omega) = \frac{\left|\tilde{V}_{src}\right|^2}{f_g\, Z_o} \qquad (6.6)$$

From (2.1), the radiated field is

$$\begin{aligned} \frac{\tilde{E}_{rad}(\omega)}{\sqrt{Z_o}} &= \frac{j\,\omega}{2\pi\, c\, r}\ \tilde{h}_N(\omega)\ \frac{\tilde{V}_{src}(\omega)}{\sqrt{f_g\, Z_o}} \\ \frac{\left|\tilde{E}_{rad}(\omega)\right|^2}{Z_o} &= \frac{1}{\lambda^2\, r^2}\ \left|\tilde{h}_N(\omega)\right|^2\ \frac{\left|\tilde{V}_{src}(\omega)\right|^2}{f_g\, Z_o} \end{aligned} \qquad (6.7)$$

Substituting (6.6) and (6.7) into (6.5), we once again find (6.4). So our expression for realized gain has been derived in two ways with the same result.

C. Antenna Gain

Antenna gain may be found from realized gain using the relationship [1]

$$\tilde{G}_r(\omega) = \tilde{G}(\omega)\left[1 - \left|\tilde{S}_{11}(\omega)\right|^2\right] \qquad (6.8)$$

where $\tilde{S}_{11}(\omega)$ is the scattering parameter looking into the antenna port as measured in a 50-ohm system. Realized gain is often a more useful measure of antenna performance than gain, because it includes the effect of impedance mismatch. It also simplifies the signal processing on a time domain antenna range, because a measurement of S_{11} is not required. Note that for well-matched antennas, the two versions of gain are very close.

D. Antenna Factor

Finally, antenna factor is expressed as

$$AF = \frac{\tilde{E}_{inc}(\omega)}{\tilde{V}_{rec}(\omega)} = \sqrt{\frac{377}{50}}\ \frac{1}{\left|\tilde{h}_N(\omega)\right|} = \frac{9.73}{\lambda\sqrt{G_r}}\,. \qquad (6.9)$$

Thus, we have shown that $h_N(t)$ and $\tilde{h}_N(\omega)$ can be related to many of the commonly used antenna descriptions in the frequency domain.

VII. A Name for $h_N(t)$

We have intentionally avoided assigning a name to $h_N(t)$ because it seemed to be controversial. In [2], Baum used "effective height" for a function related to $h_N(t)$, so some variation on this term might seem a natural choice.

However, there are two issues with using some variation of "effective height" for $h_N(t)$. First, there are already several alternative meanings of "effective height," according to [1]. Adding one more definition might add to the confusion. Second, the units of $h_N(t)$ are meters/second in the time domain, and meters in the frequency domain. Since we are developing this function primarily to describe performance in the time domain, the units seem mismatched.

Our preferred name for $h_N(t)$ is "antenna impulse response." This might seem surprising since $h_N(t)$ is proportional to the responses to a step voltage in transmission, and to an impulse field in reception. While we understand why this might cause confusion, it reflects the way that antennas behave. An antenna in transmission responds not to the source voltage, but to its time derivative.

The default assumption is that "antenna impulse response" refers to a 50 Ω system. When used to describe antenna performance in other impedance systems, the terminology would be, for example, "antenna impulse response in a 75 Ω system." The relationship between the two is derived in Section IV.

VIII. Conclusion

We have developed a function for describing antenna response, $\vec{h}_N(\theta,\phi,t)$; or on boresight for dominant polarization, $h_N(t)$. We have shown how it describes antenna performance as simply as possible, in both transmission and reception, and in both the frequency and time domains. We have shown that other commonly used antenna parameters, such as gain, realized gain, and antenna factor, can be simply expressed in terms of this function. Finally, we presented our argument for why this function should be called "antenna impulse response," and why that term should be adopted as standard terminology by the IEEE.

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